به‌نام خدا

سیستم‌های شاسی و بدنه خودرو

سیستم هدایت و فرمان

دوره کارشناسی ارشد مهندسی خودرو
دانشگاه علم و صنعت ایران

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Steering

Vehicles classified according to how their path is controlled:

- *Guided vehicles (kinematically guided vehicles), trajectory* is fixed by kinematic constraints.
- *Piloted vehicles, the trajectory, a tri-dimensional or planar curve,* is determined by a guidance system controlled by a human pilot or by a device, usually electro-mechanical.

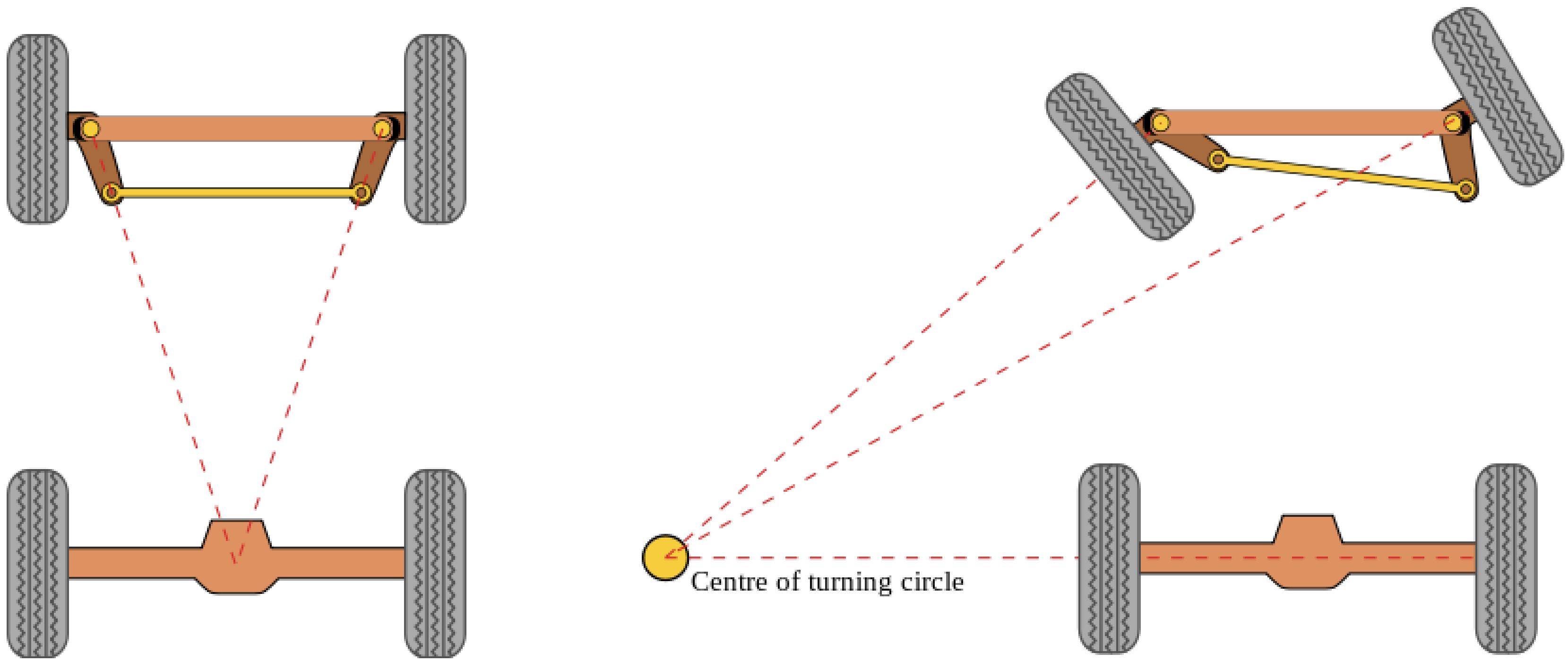
Steering

The main components of the steering system include:

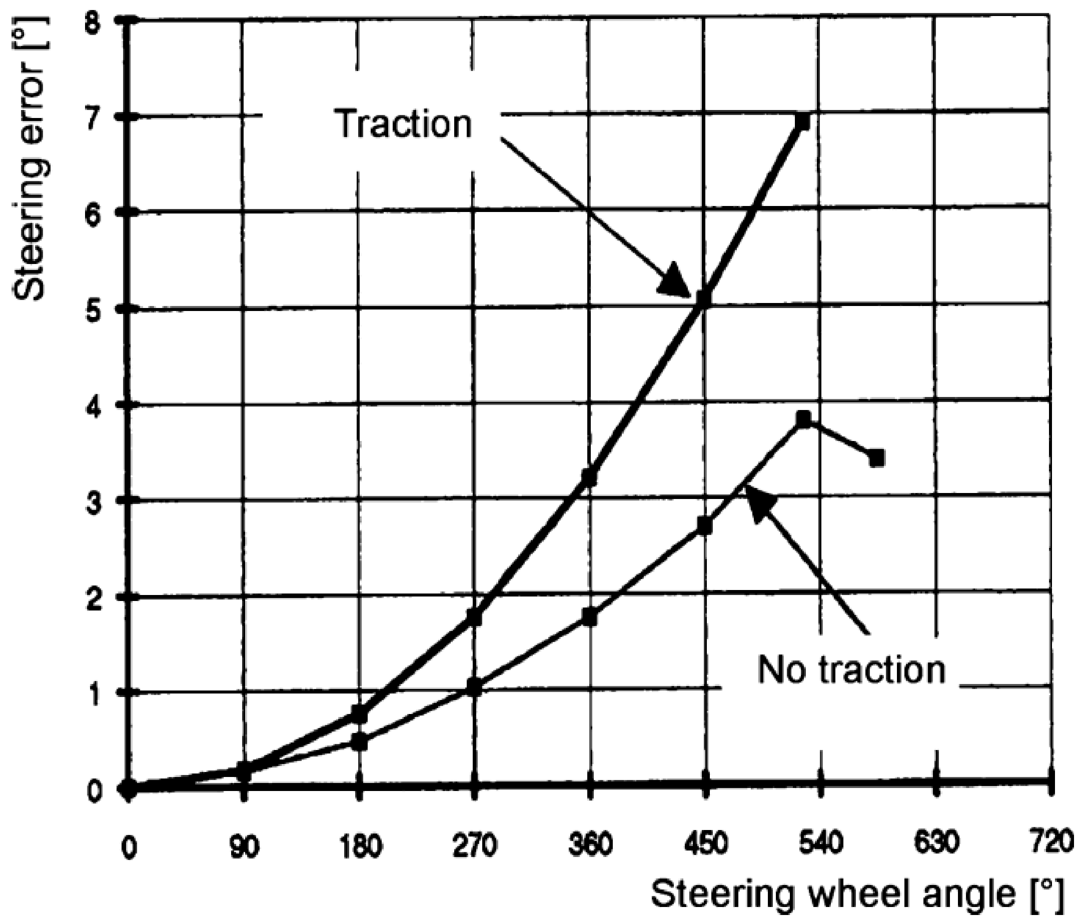
- *Steering mechanism, the system of linkages steering the front wheels in a* particular way around the king-pin axis, connecting steering arms moving with the suspension stroke to the steering box
- *Steering box, transforming steering wheel rotation into a displacement of* the steering tie rods
- *Steering column, connecting steering wheel with steering box*

Steering System

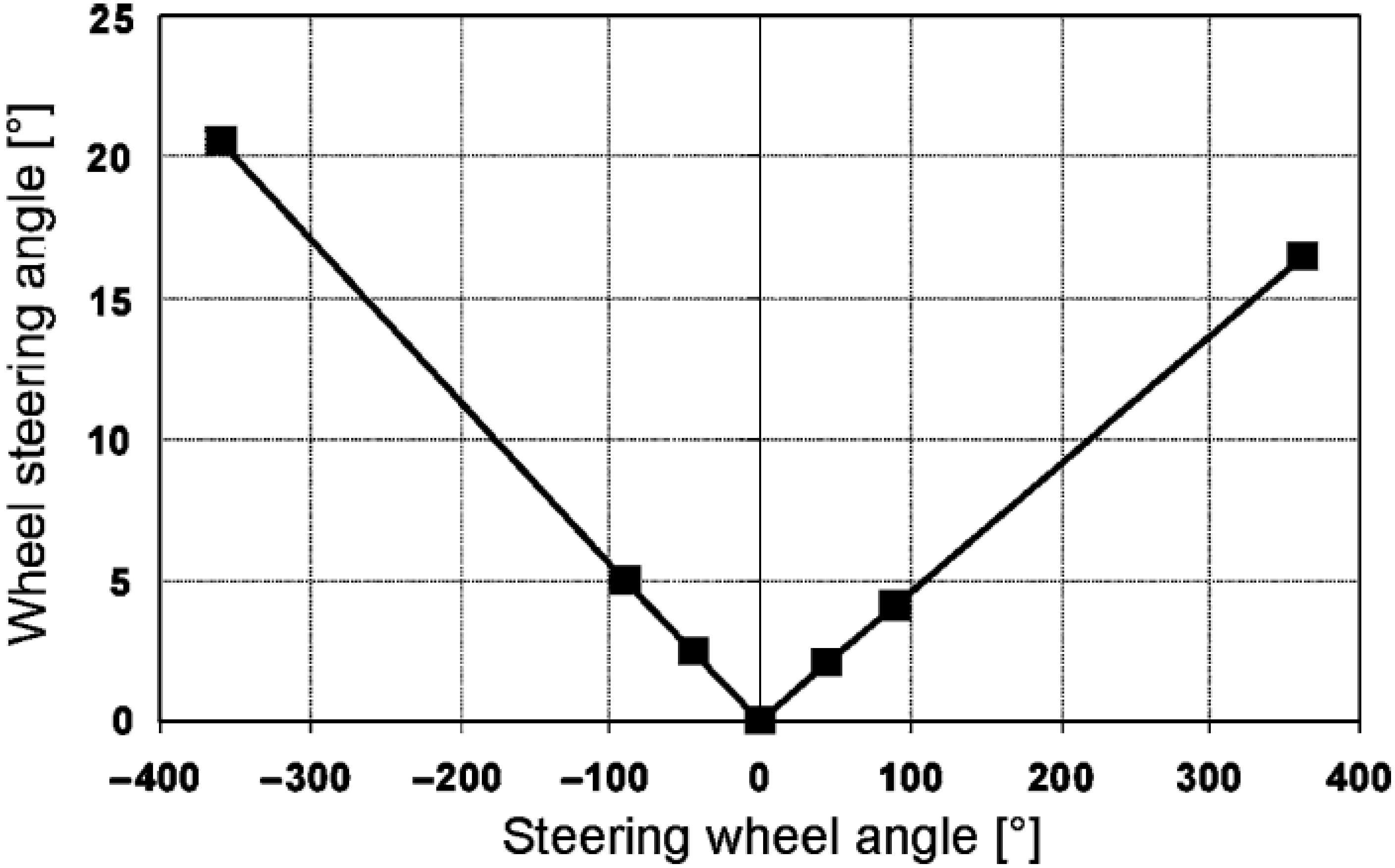
The Jeantaud's condition could only be verified if tie rods and rack are aligned and the steering arms cross in the middle of the rear axle center line.



Steering System



Steering System

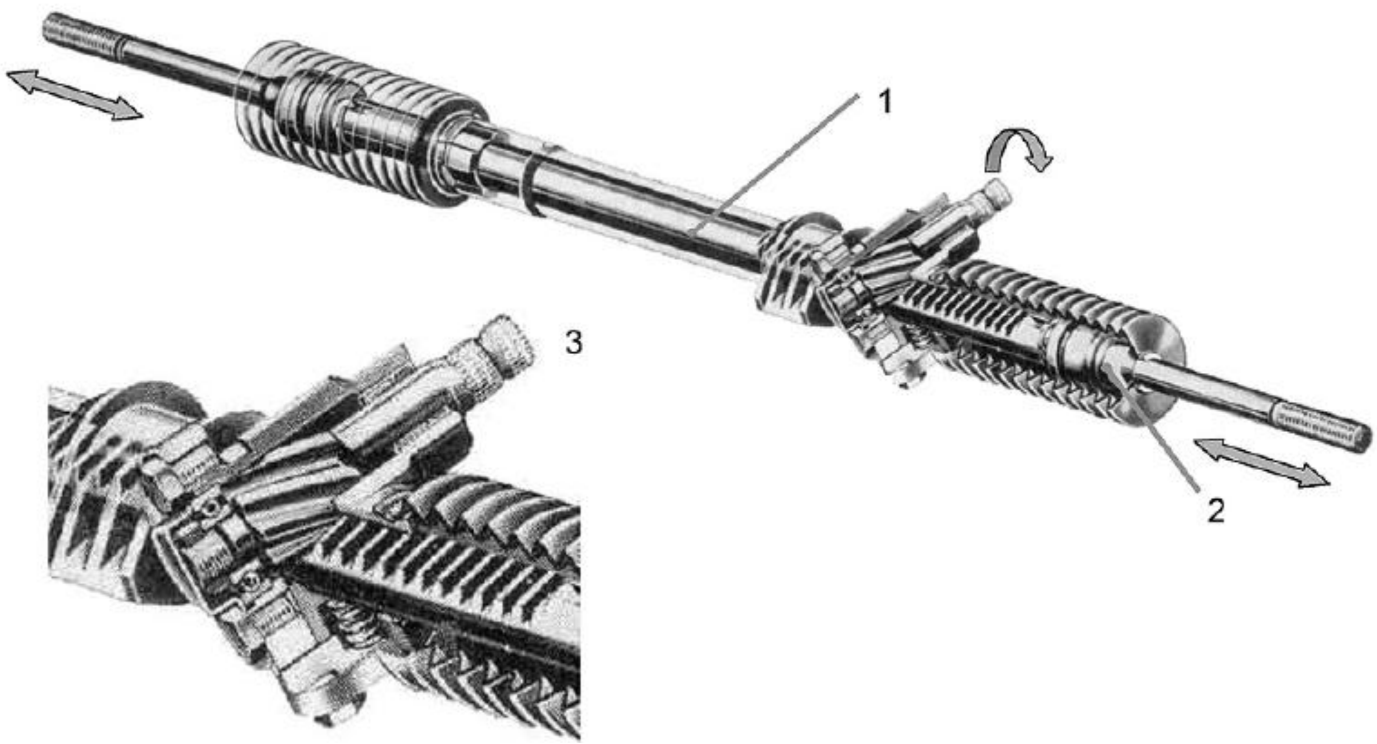


RACK AND PINION STEERING BOX

- A rack and pinion steering box is found today in almost all cars and light duty industrial vehicles.
- The alternative, the screw and sector steering box and its variants, is in practice reserved for heavy duty industrial vehicles or those offroad vehicles still featuring a rigid front axle.

RACK AND PINION STEERING BOX

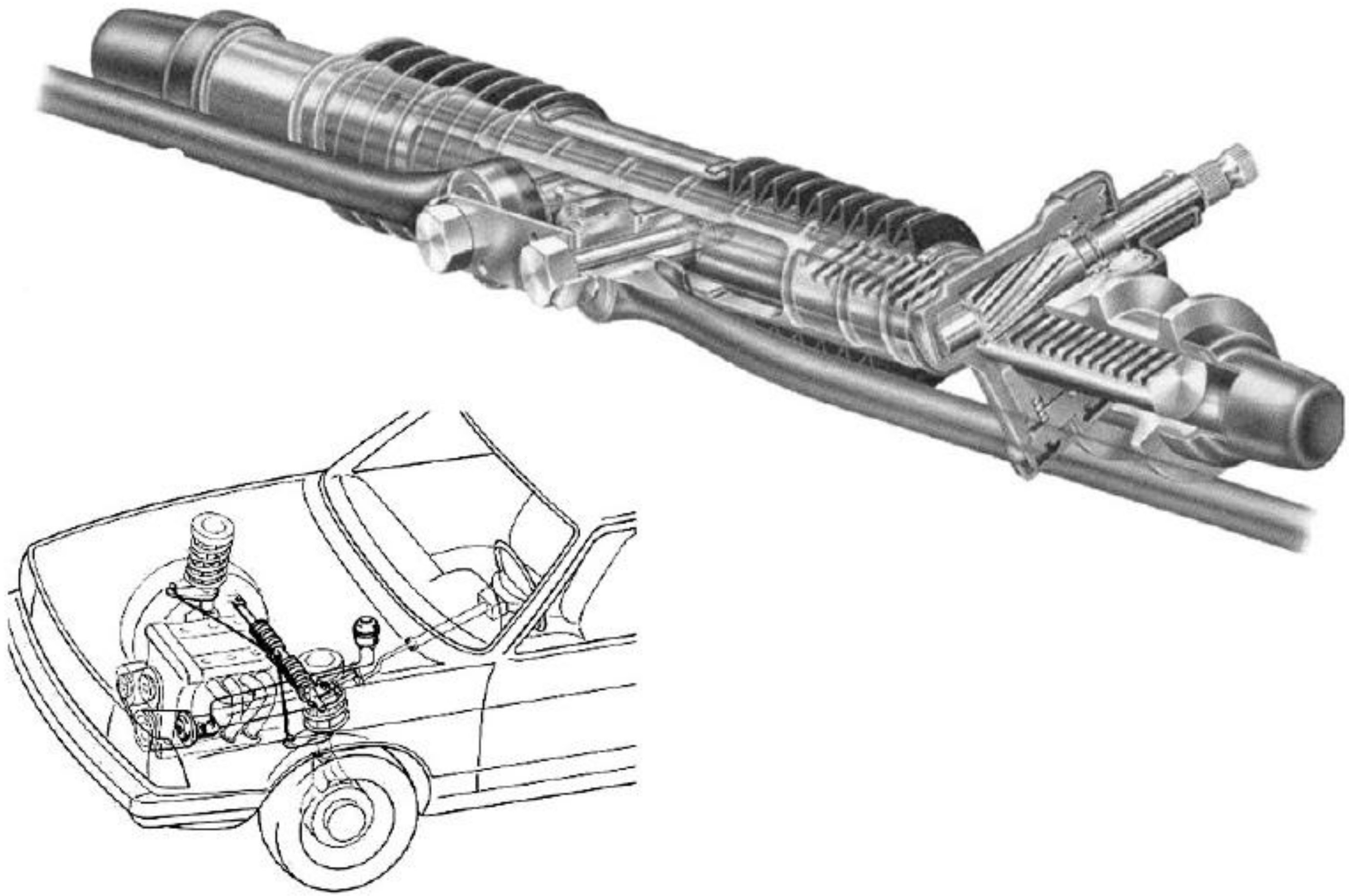
- The device transforms, through the geared couple of the pinion 3 and the rack 1, the rotary motion of the steering wheel, applied by the driver, into a linear motion of the spheric heads 2, which operate the steering mechanism.



RACK AND PINION STEERING BOX

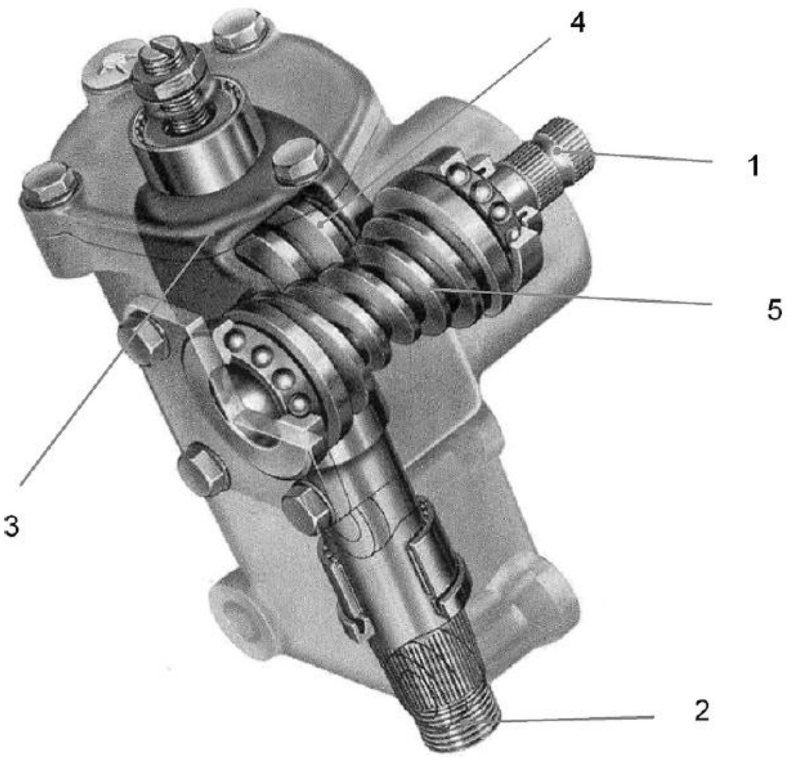
- Because of the simplicity of the mechanism and the reduced friction between the teeth flanks, mechanical efficiency is usually excellent
- This fact is helpful because it reduces the reaction torque on the steering wheel and gives the driver a true and accurate feeling of the existing lateral tire-road friction.
- As a disadvantage, the steering transmission ratio cannot be increased beyond certain values because it is limited by tooth size. The

RACK AND PINION STEERING BOX

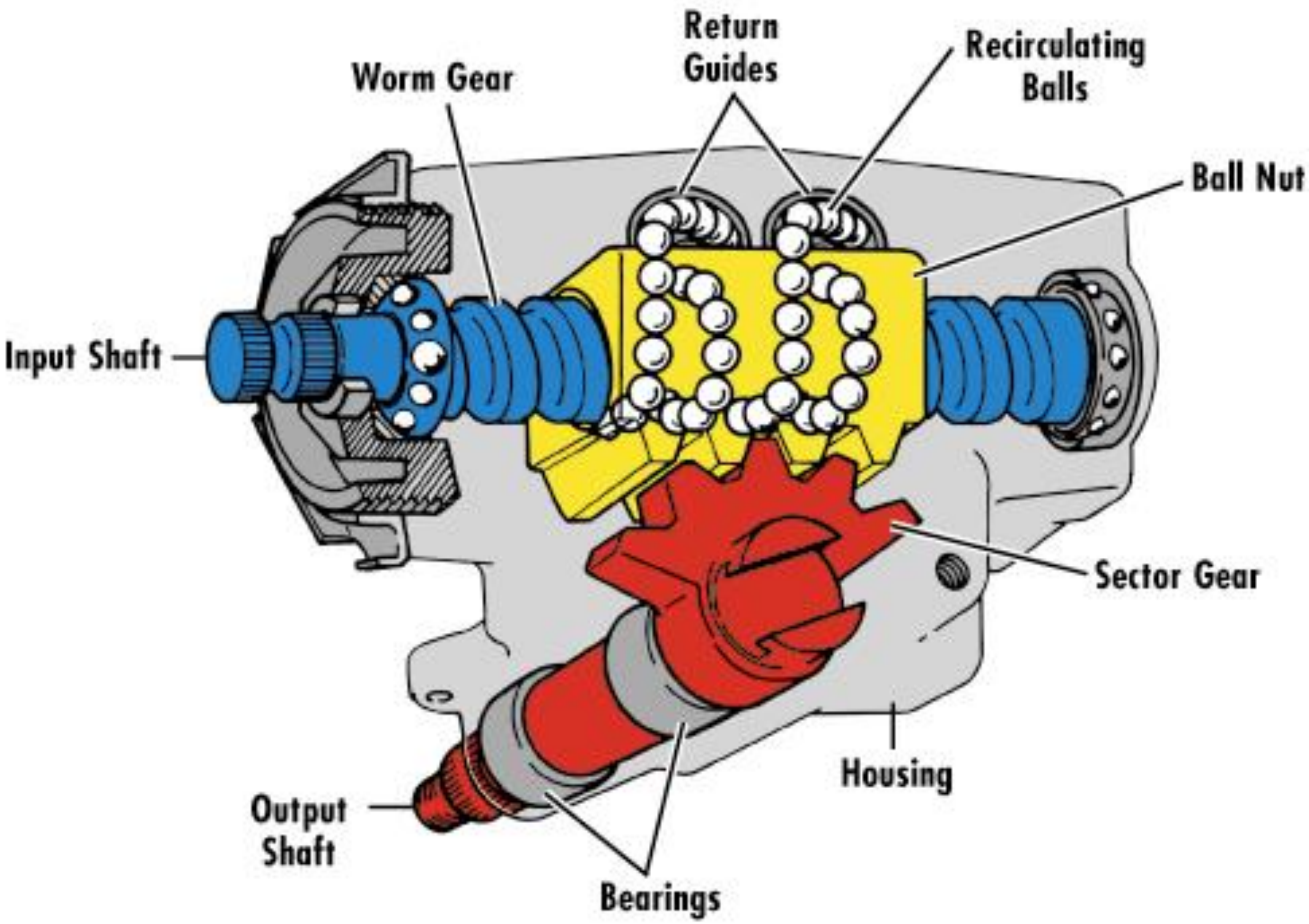


SCREW AND SECTOR STEERING BOX

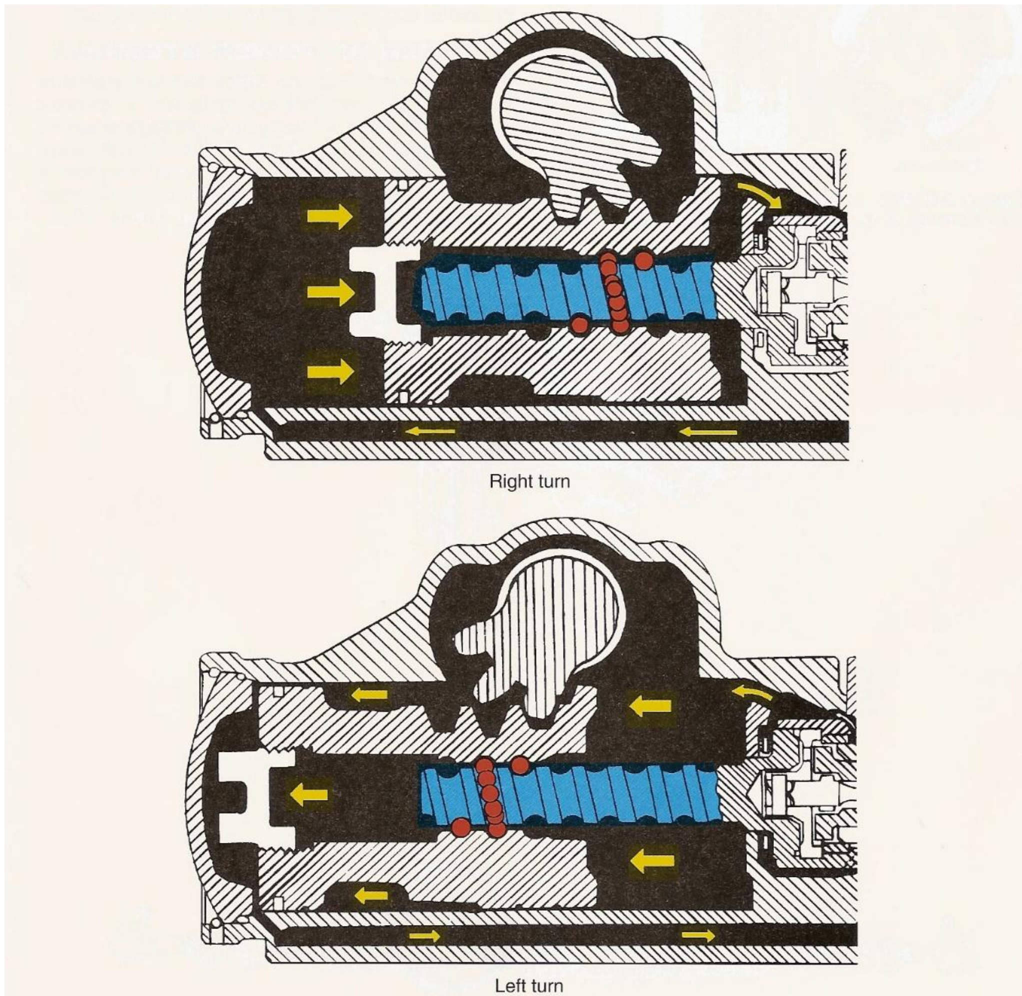
Screw and sector steering boxes are installed, as we have explained, primarily on industrial vehicles with either rigid or independent suspensions



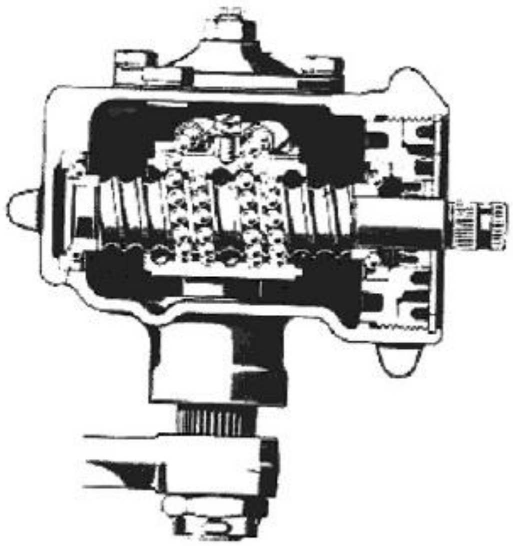
Recirculating Ball Screw

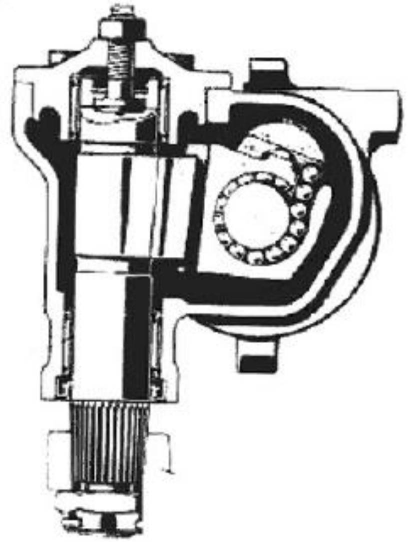


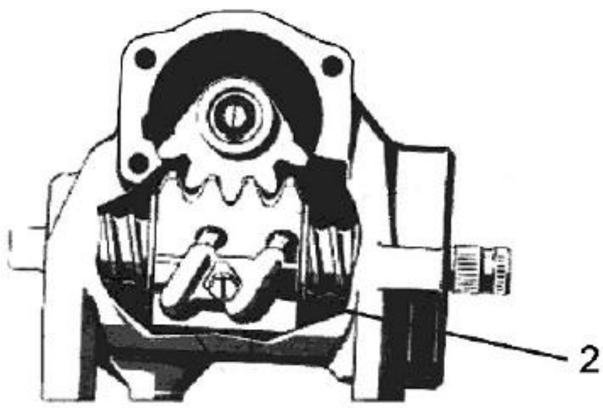
Recirculating Ball Screw

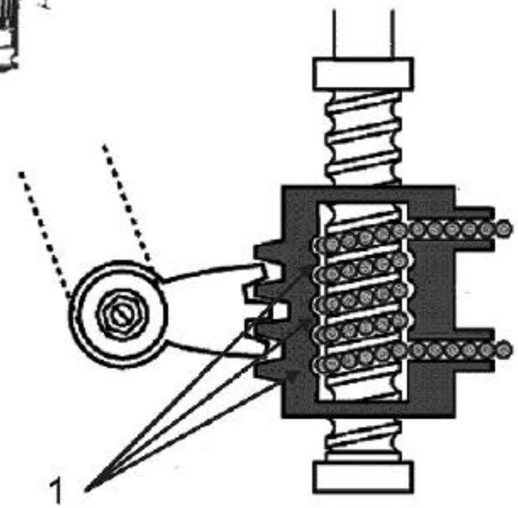


Recirculating Ball Screw

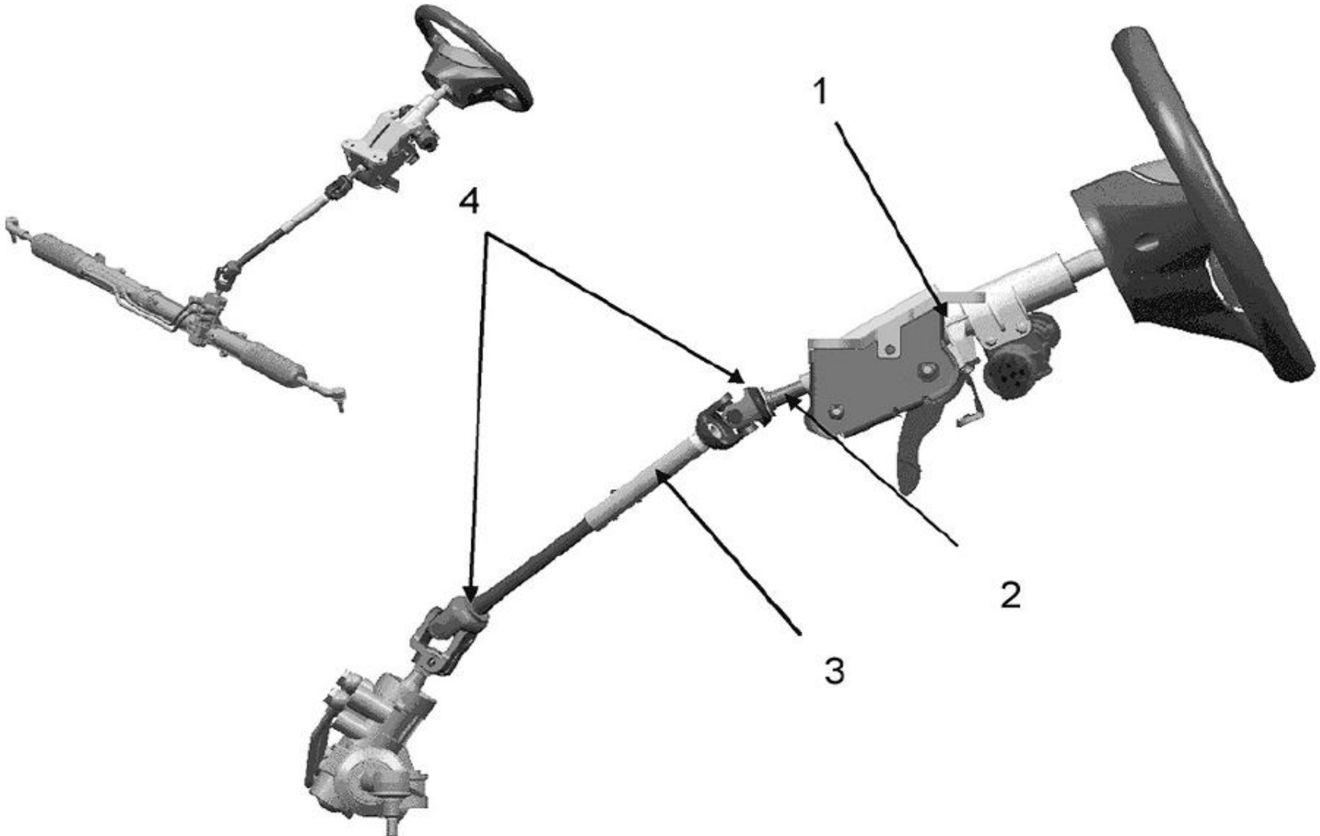








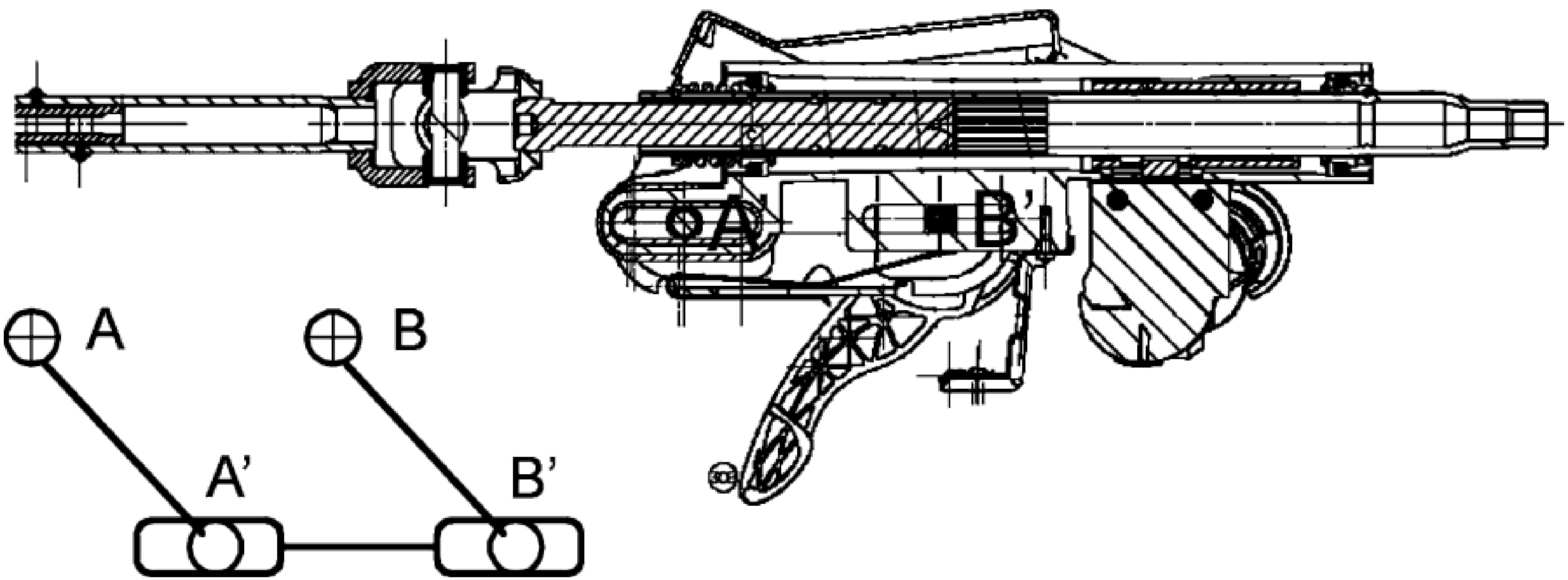
STEERING COLUMN



This component transfers the torque applied to the steering wheel by the driver to the steering box. Seldom do steering box and steering wheel position in modern cars allow to have a straight steering column; for this reason the column is made up of three sections.

STEERING COLUMN

Adjustable structure



POWER STEERING

- The general mass increase in all cars has affected loads between tires and ground and, therefore, the torque to be applied to the steering box to steer the front wheels.
- The reduction of steering wheel torque to levels that are ergonomically acceptable has been obtained by applying power assistance to the existing mechanical systems

POWER STEERING

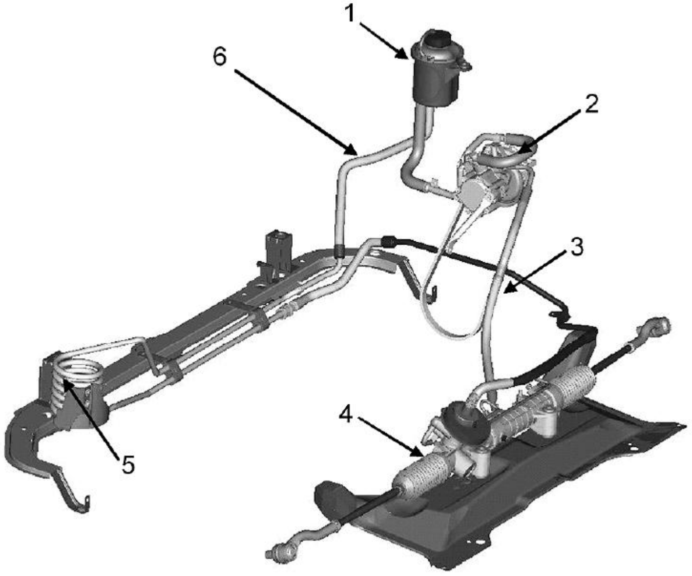
- The most widely used power assistance is hydraulic, sometimes integrated by an electro-hydraulic device with electronic control, to adjust the effect to vehicle speed.

POWER STEERING

Hydraulic rack and pinion steering box

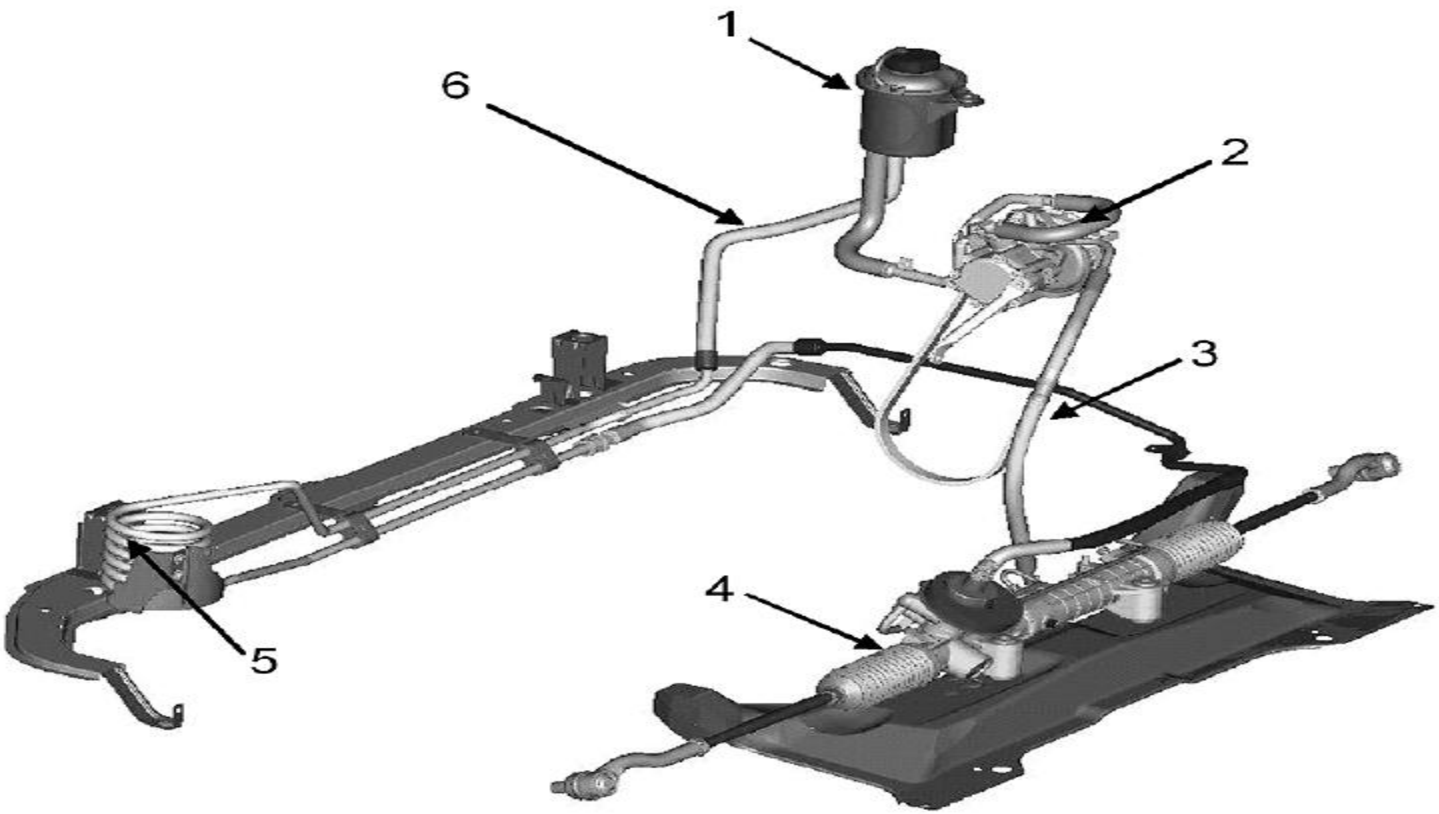
- The system includes the following components:

1. Oil reservoir
2. Pump, normally blade type, driven by the accessory belt of the engine
3. High pressure tube
4. Steering box
5. Cooling serpentine
6. Low pressure return tube



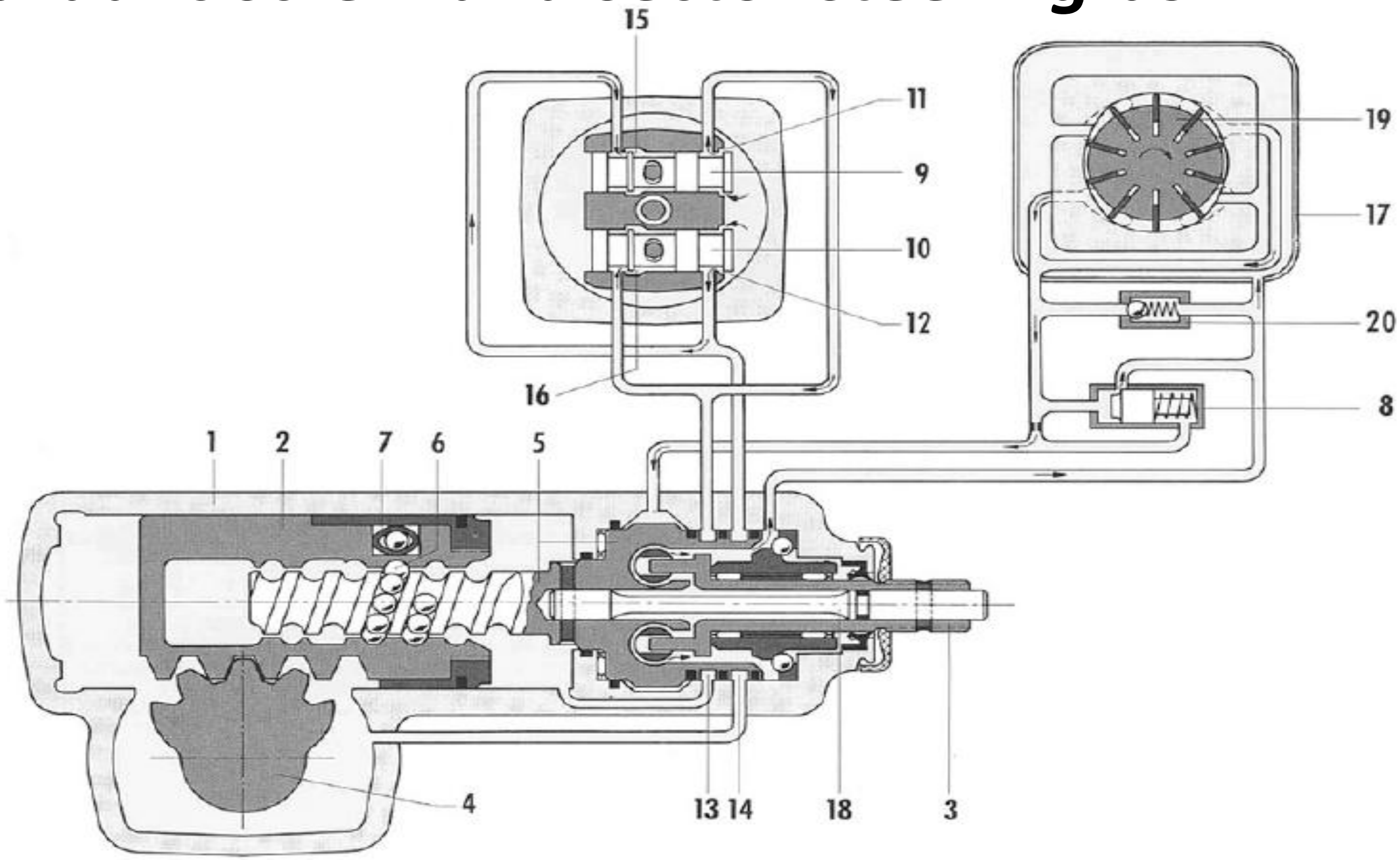
POWER STEERING

Hydraulic rack and pinion steering box



POWER STEERING

Hydraulic screw and sector steering box

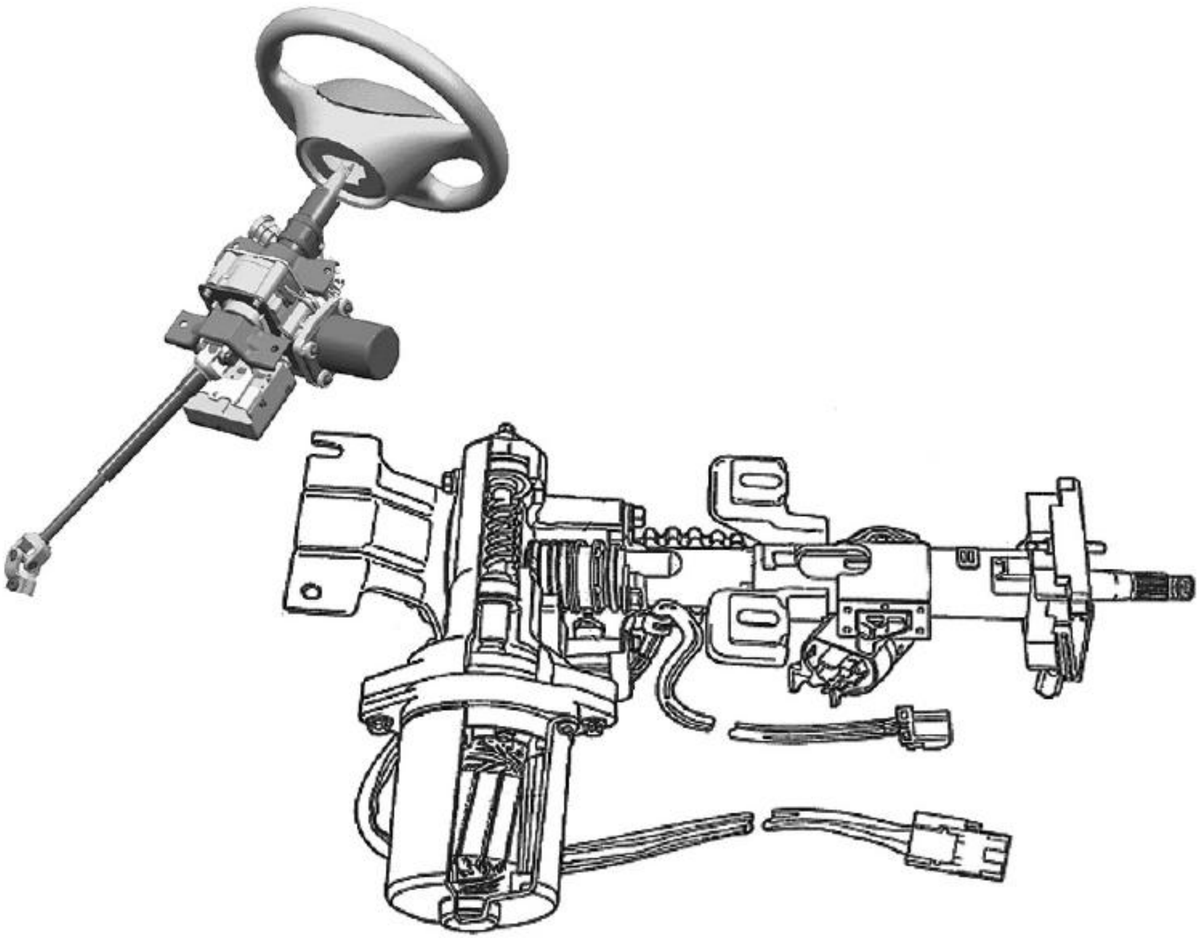


ELECTRIC POWER STEERING

Electric power assistance

- In small and medium size cars electric power steering (EPS) systems have become widespread. They are applied to rack and pinion steering boxes and operate, substantially, through an electric motor adding torque to the steering column, or to the pinion directly.
- The electric motor is regulated by an electronic controller, which again has the function of generating an assistance torque proportional to the steering torque.
- The control system includes sensors capable of measuring steering torque, vehicle speed, steering wheel speed and angle.

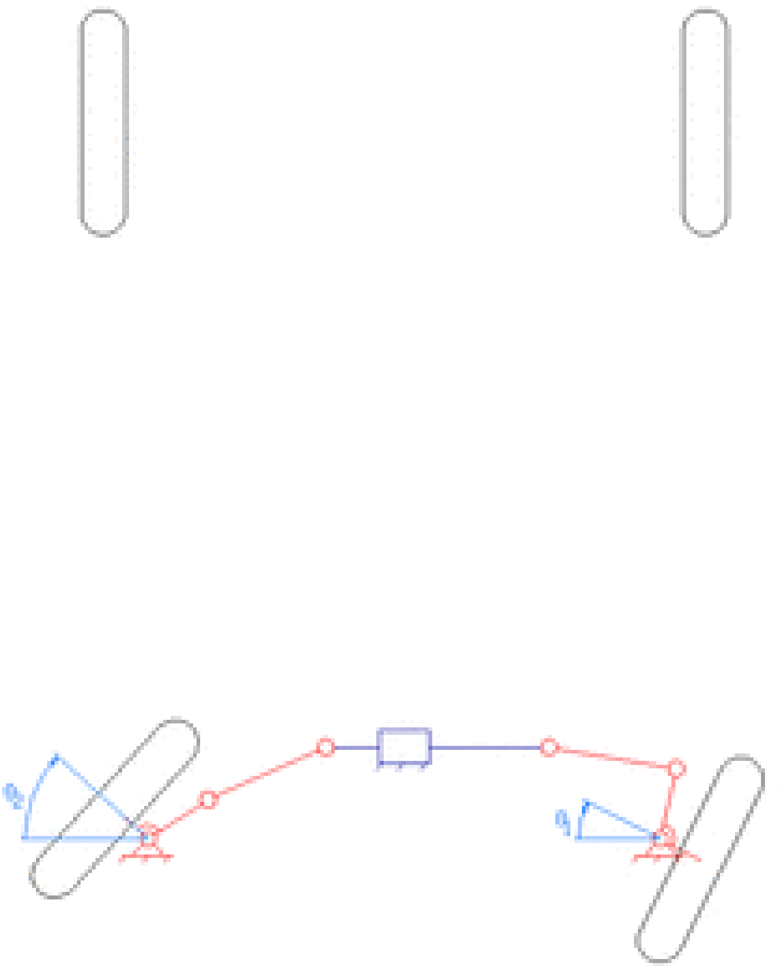
ELECTRIC POWER STEERING



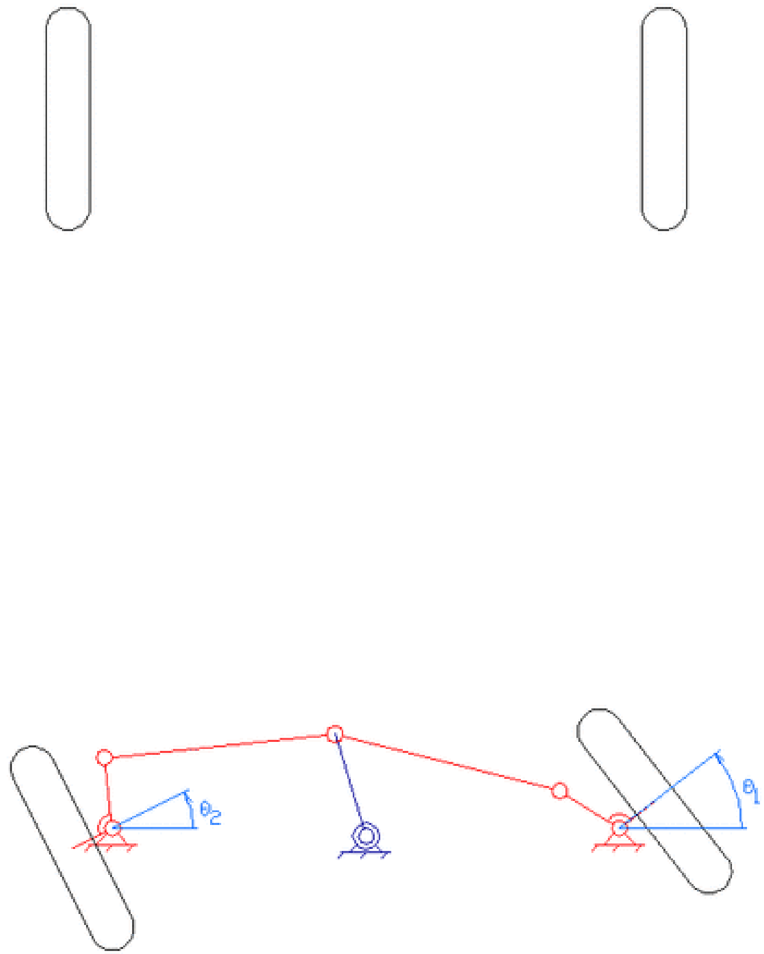
ELECTRIC POWER STEERING

- This system has a number of advantages when compared with a conventional hydraulic unit:
- *Oil circulation is eliminated, with consequent simplification of the engine* compartment lay-out.
- *Fuel consumption is reduced, because torque control is not based upon* wasted power.
- *Active safety is increased, because assistance is also available when the* engine stalls or is switched off.
- *Possibility of more sophisticated torque control, as found in hydraulic systems.*
- *Low impact on design of mechanical components.*

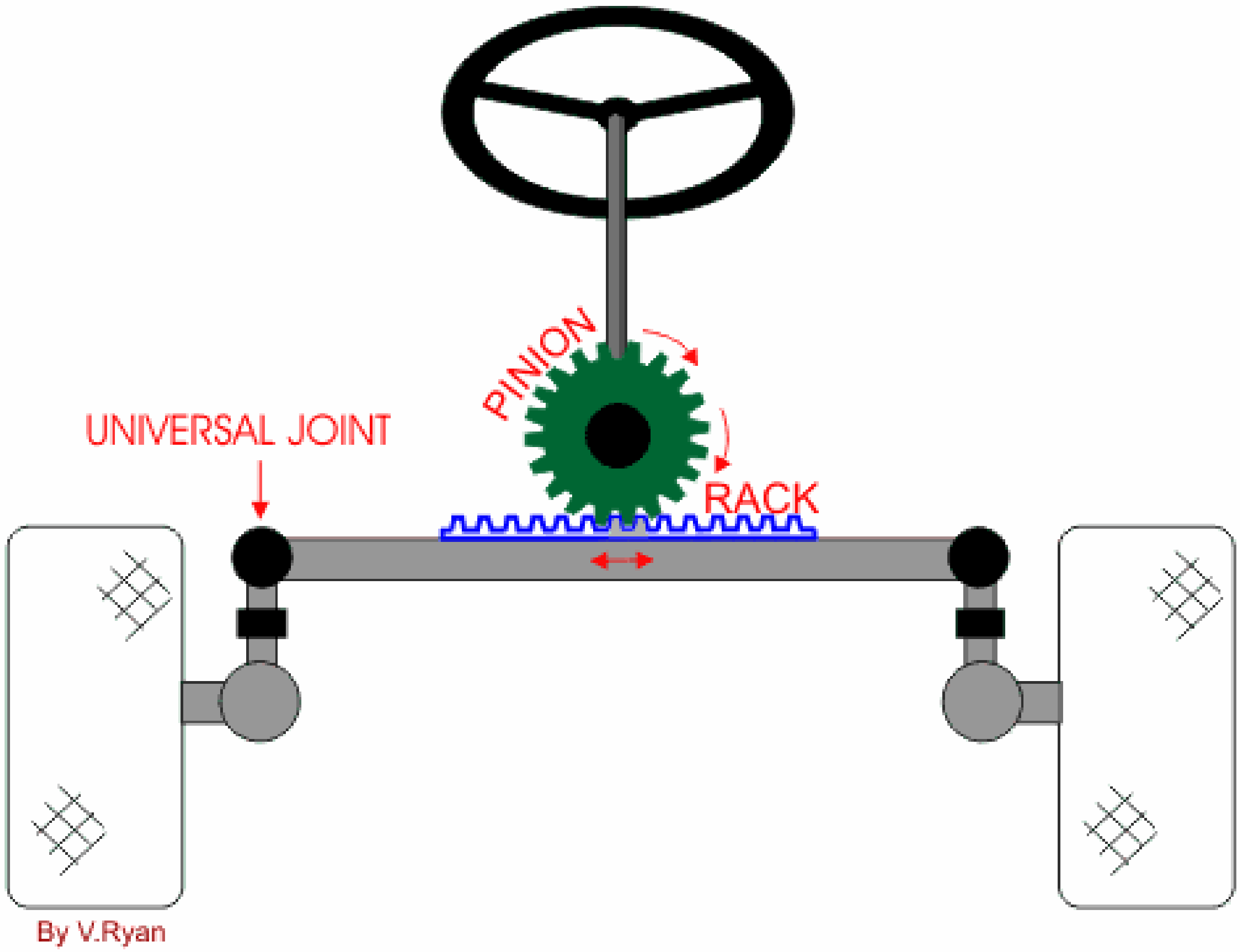
STEERING MECHANISMS



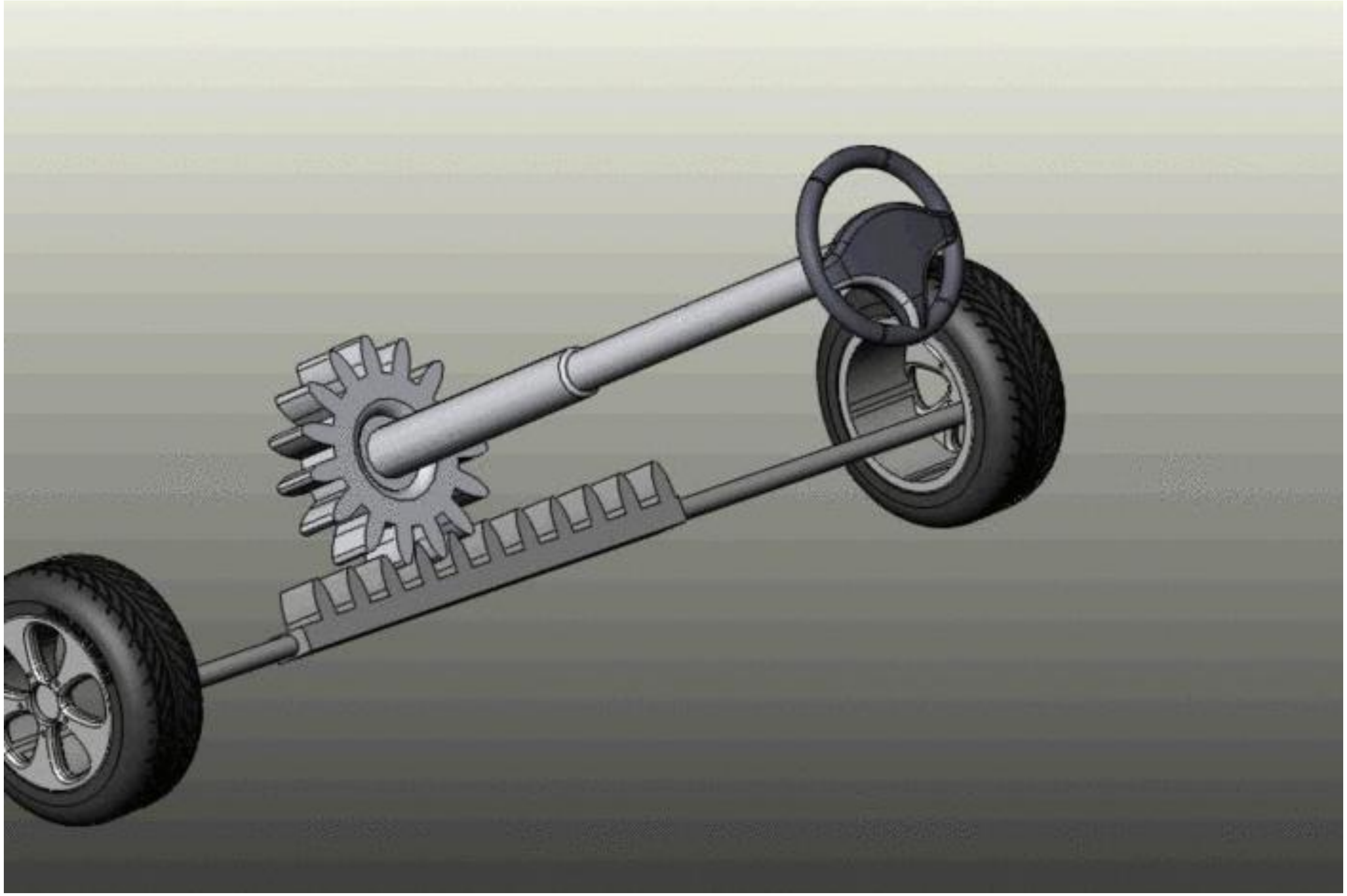
STEERING MECHANISMS



STEERING MECHANISMS



STEERING MECHANISMS



STEERING MECHANISMS

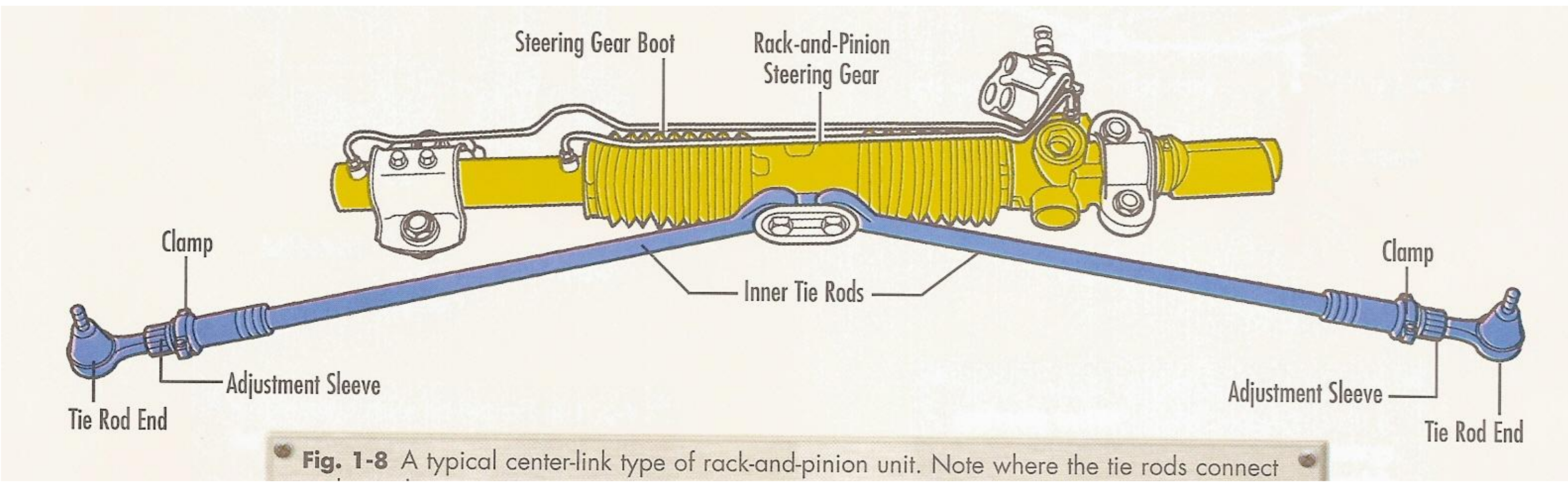
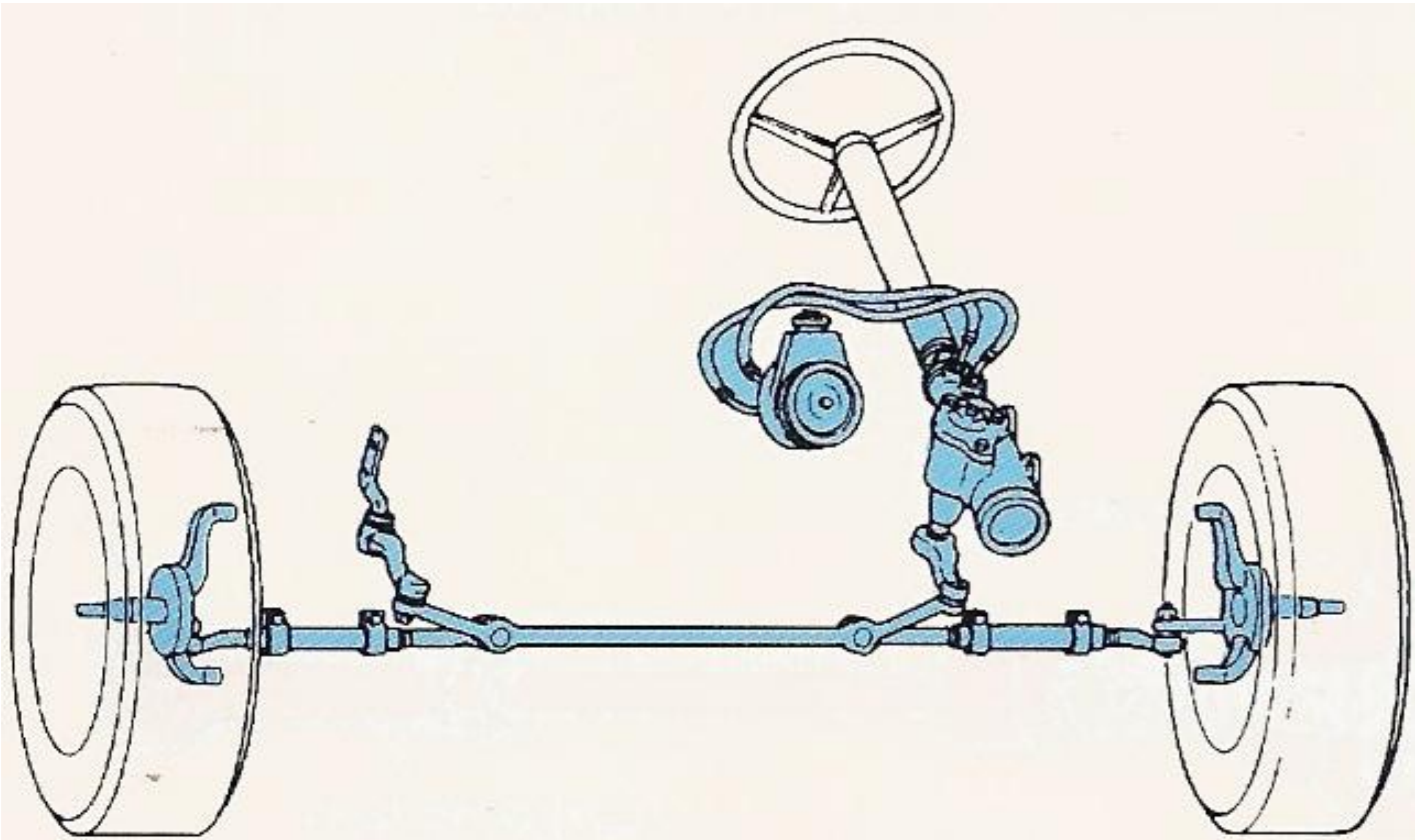
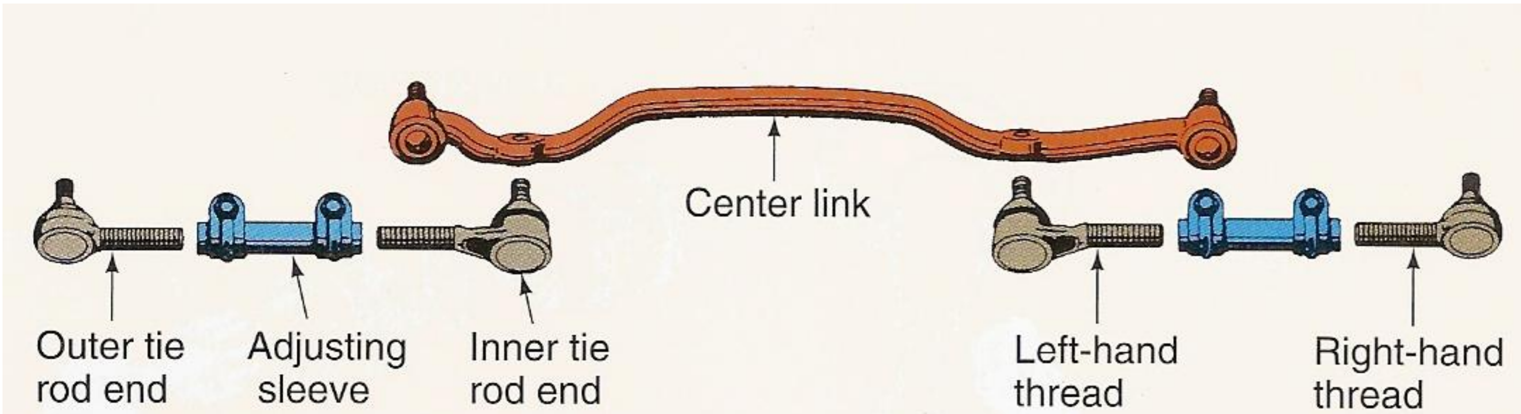


Fig. 1-8 A typical center-link type of rack-and-pinion unit. Note where the tie rods connect

STEERING MECHANISMS



STEERING MECHANISMS



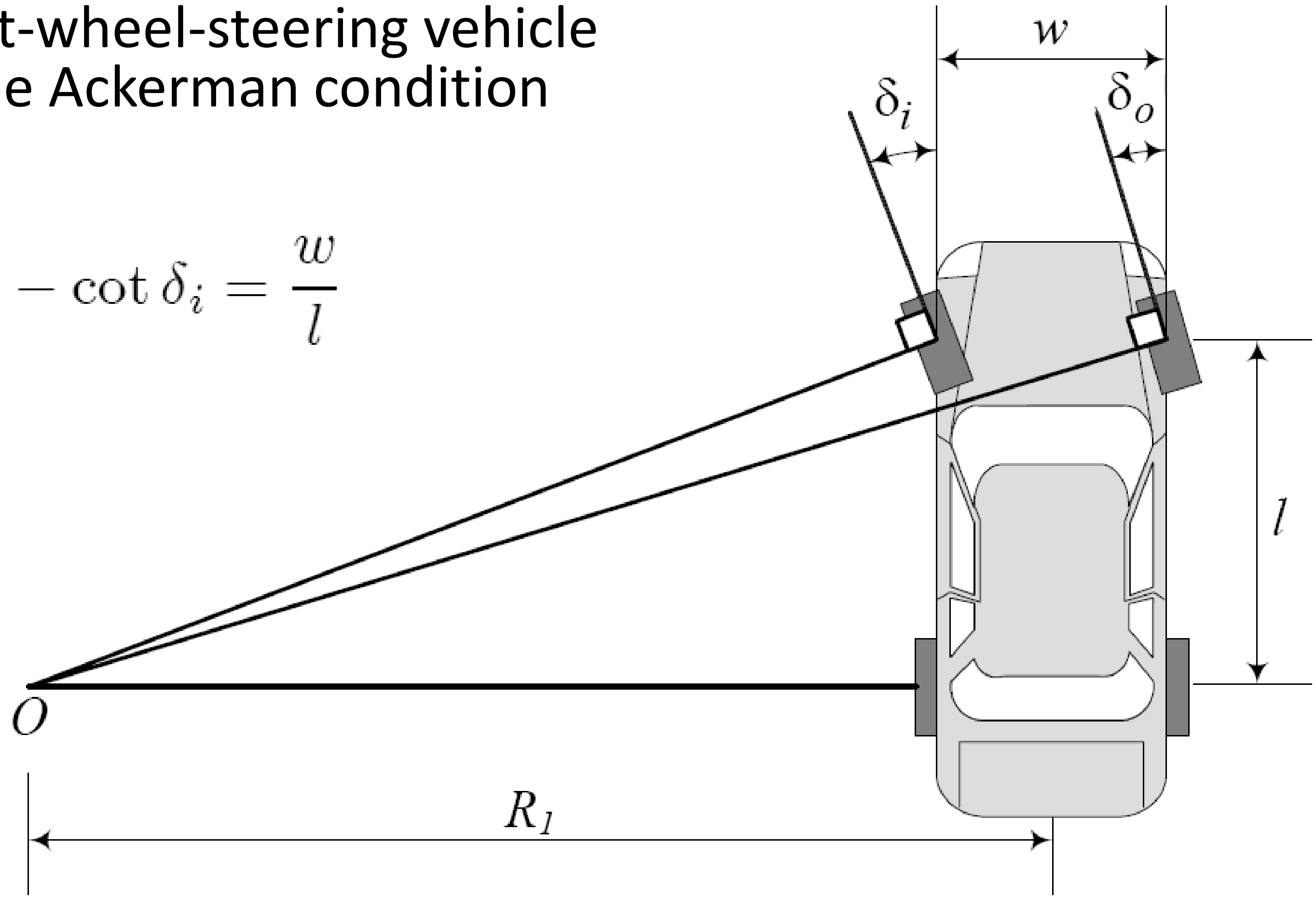
Idler Arm

Pitman Arm

Kinematic *Steering*

A front-wheel-steering vehicle
and the Ackerman condition

$$\cot \delta_o - \cot \delta_i = \frac{w}{l}$$



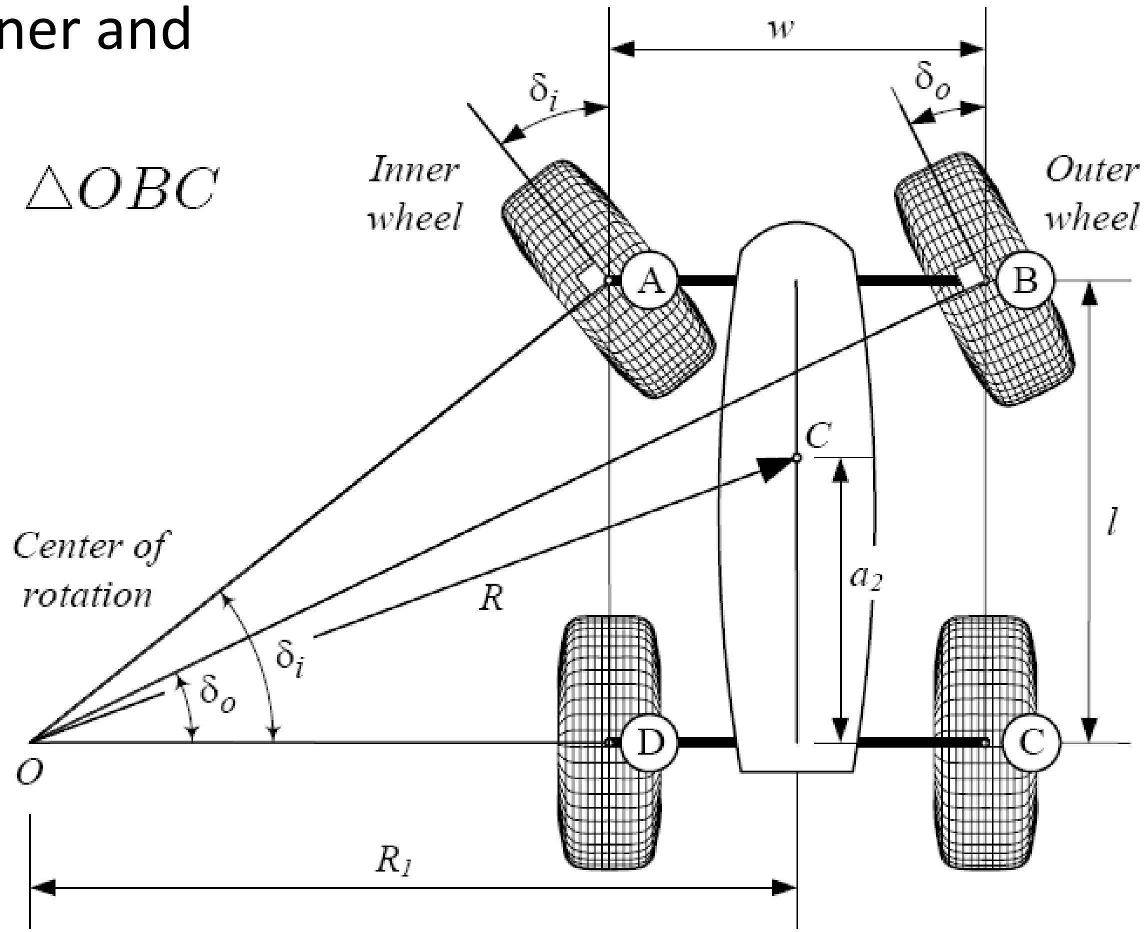
Kinematic *Steering*

Steer angles of the inner and outer wheels

triangles $\triangle OAD$ and $\triangle OBC$

$$\tan\delta_i = \frac{l}{R_1 - \frac{w}{2}}$$

$$\tan\delta_o = \frac{l}{R_1 + \frac{w}{2}}$$

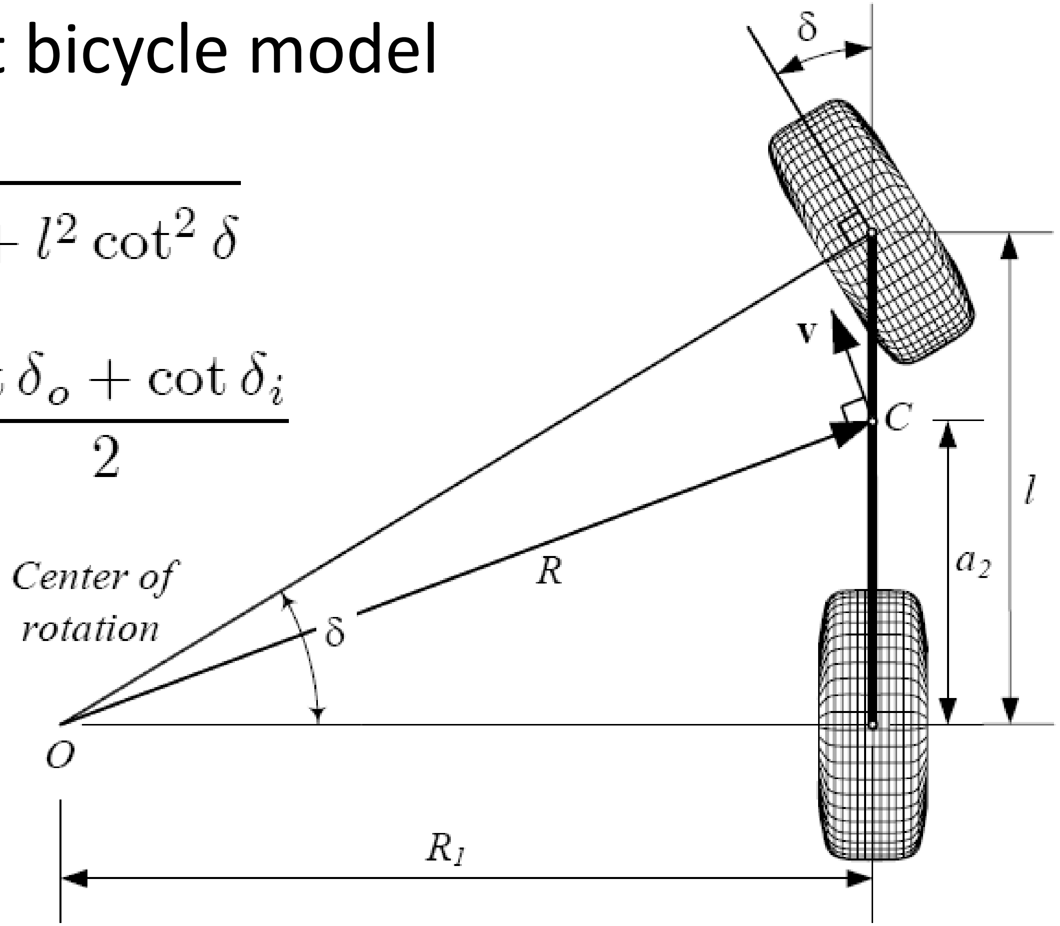


Kinematic *Steering*

Equivalent bicycle model

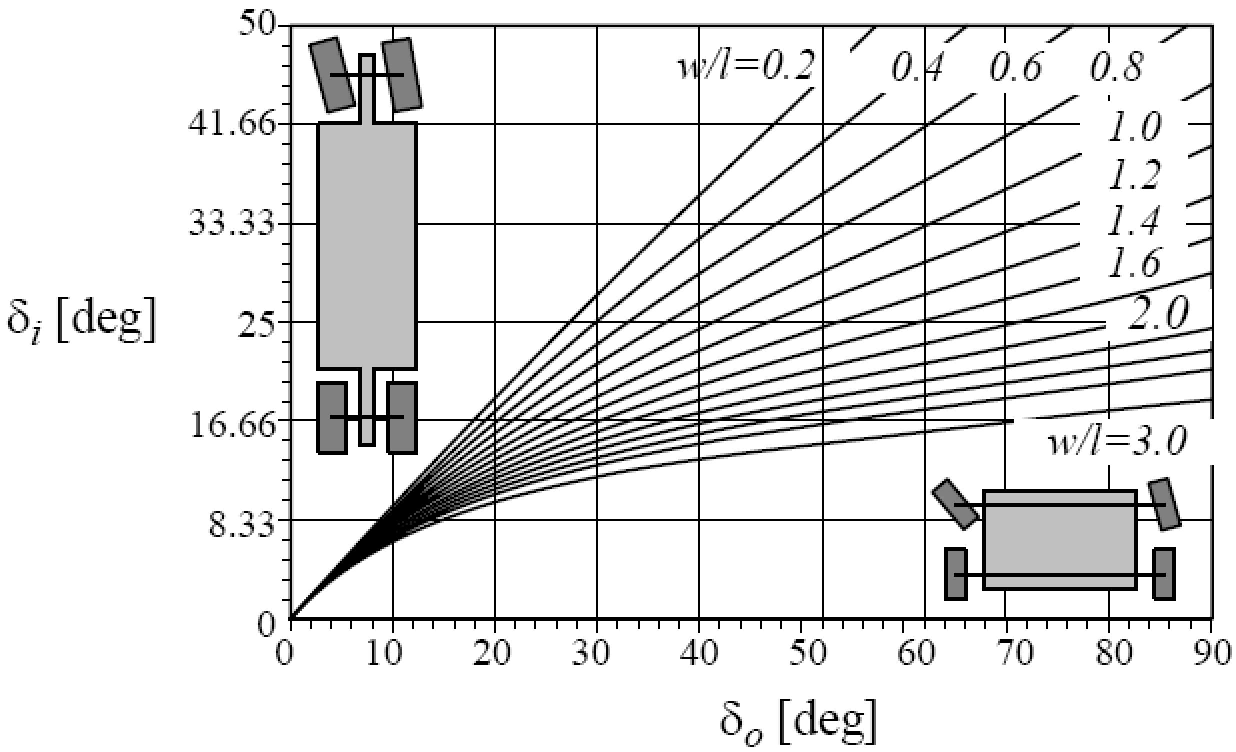
$$R = \sqrt{a_2^2 + l^2 \cot^2 \delta}$$

$$\cot \delta = \frac{\cot \delta_o + \cot \delta_i}{2}$$



Kinematic *Steering*

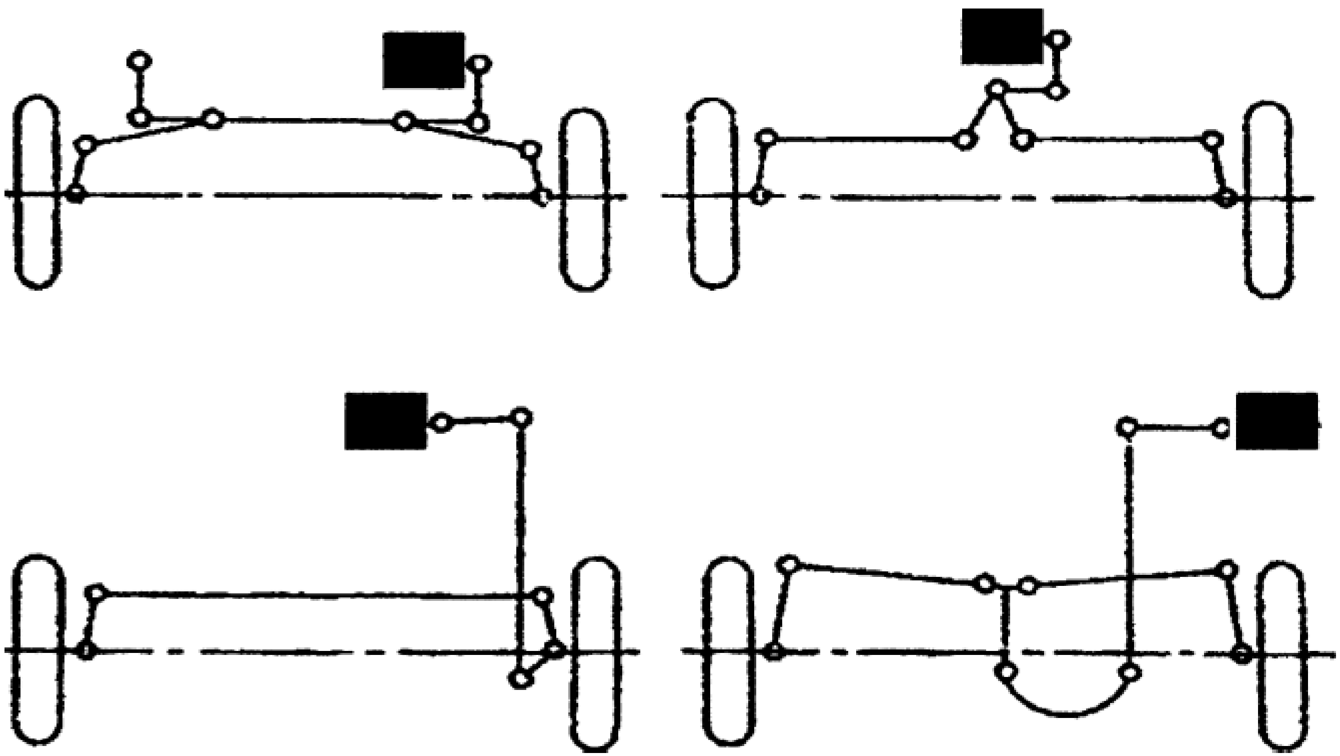
Effect of w/l on the Ackerman condition



Steering System

•Although the importance of correct kinematic steering is often overestimated, steering error has a significant effect on tire wear of the front wheels and on steering wheel self-alignment.

•Instead of the track*, it is better that the above equation contain the distance between the kingpin* axes of the wheels, or between their intersections with the ground.



Steering System

Monotrace model:

- Radius of the trajectory of the centre of mass of the vehicle
- Steering angle of the equivalent two-wheeled vehicle

If the radius of the trajectory is large compared to the wheelbase of the vehicle:

$$R \approx l\cot(\delta) \approx l/\delta$$

or:

$$1/R\delta \approx 1/l$$

1/*Rδ has an important physical meaning: It is the ratio* between the response of the vehicle, in terms of curvature 1/*R of the trajectory*, and the input *δ* which causes it. It is, therefore, a sort of transfer function for the directional control (trajectory curvature gain).

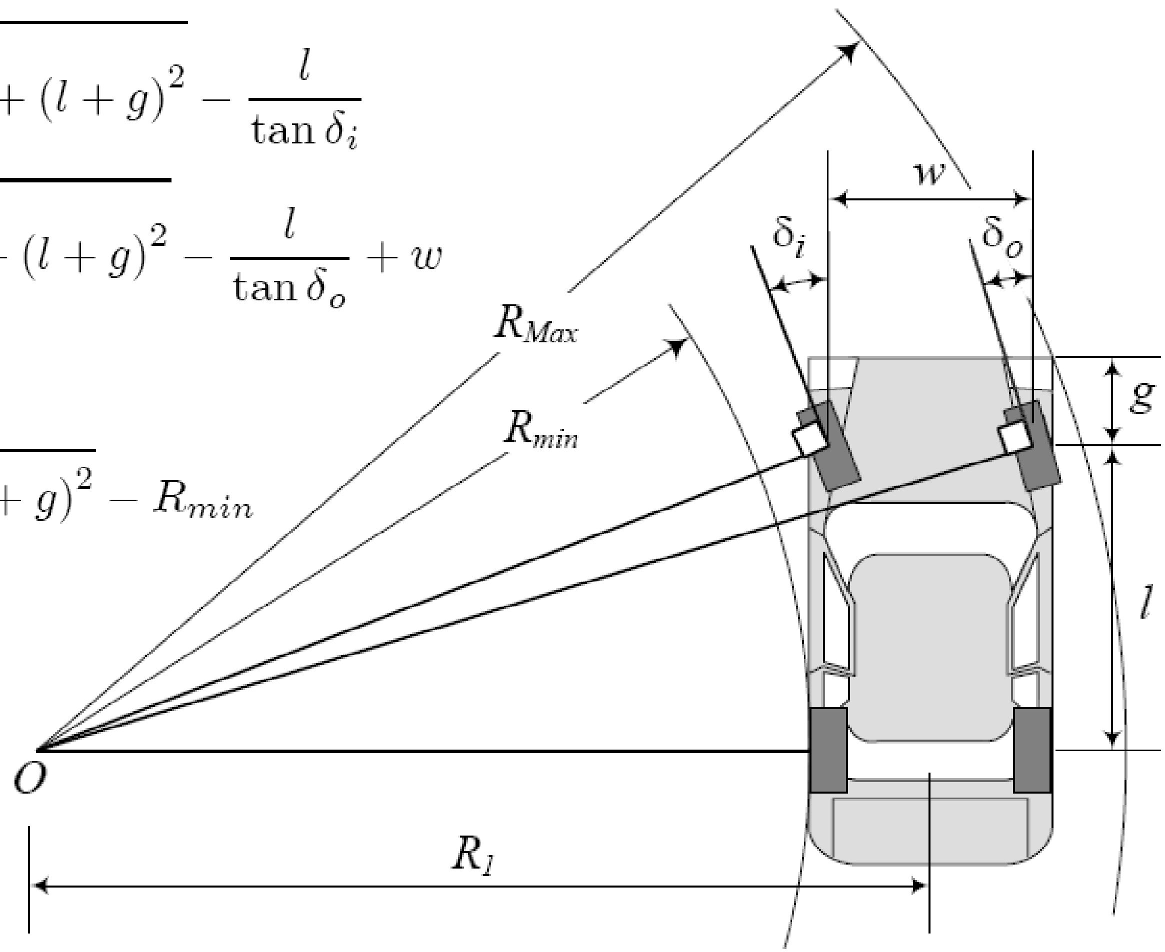
Kinematic *Steering*

Required space for a turning two-axle vehicle.

$$\begin{aligned}\triangle R &= \sqrt{\left(\frac{l}{\tan\delta_i}+2w\right)^2+(l+g)^2}-\frac{l}{\tan\delta_i}\\ &= \sqrt{\left(\frac{l}{\tan\delta_o}+w\right)^2+(l+g)^2}-\frac{l}{\tan\delta_o}+w\end{aligned}$$

$$\begin{aligned}\triangle R &= R_{Max}-R_{min}\\ &= \sqrt{(R_{min}+w)^2+(l+g)^2}-R_{min}\end{aligned}$$

$$\begin{aligned}R_{min} &= R_1-\frac{1}{2}w\\ &= \frac{l}{\tan\delta_i}\\ &= \frac{l}{\tan\delta_o}-w\end{aligned}$$



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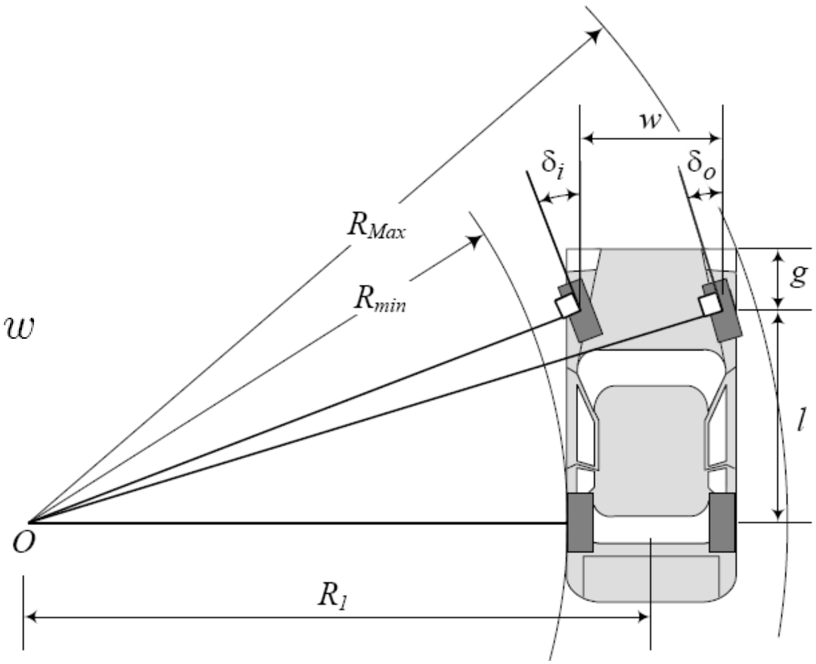
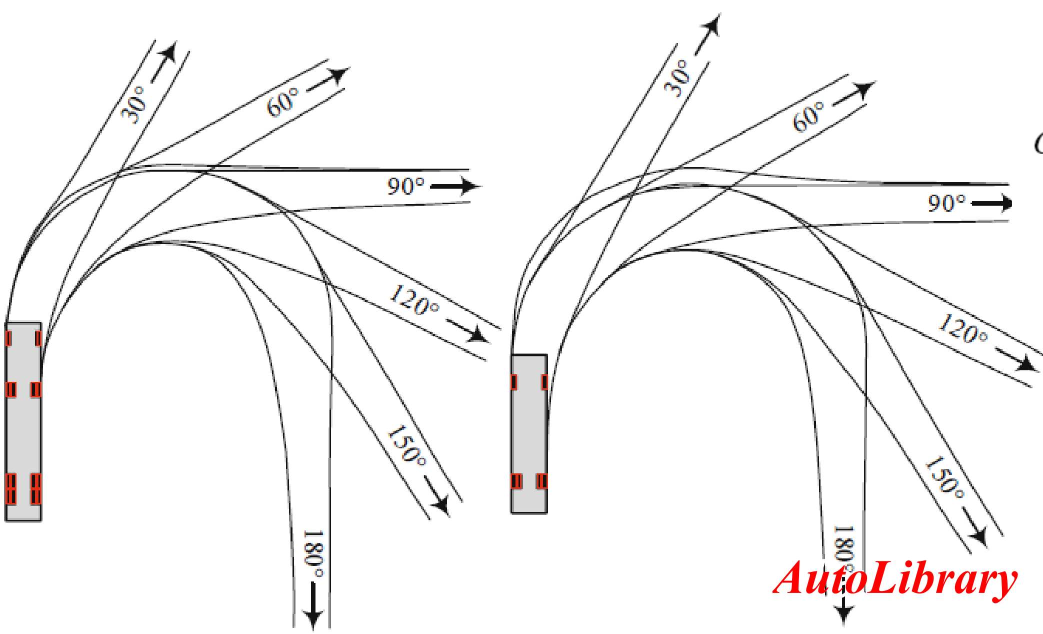
Kinematic *Steering*

The requirement for more space becomes more critical when the vehicle is longer and w/l increases.

$$\triangle R = \sqrt{\left(\frac{l}{\tan\delta_i} + 2w\right)^2 + (l+g)^2} - \frac{l}{\tan\delta_i}$$

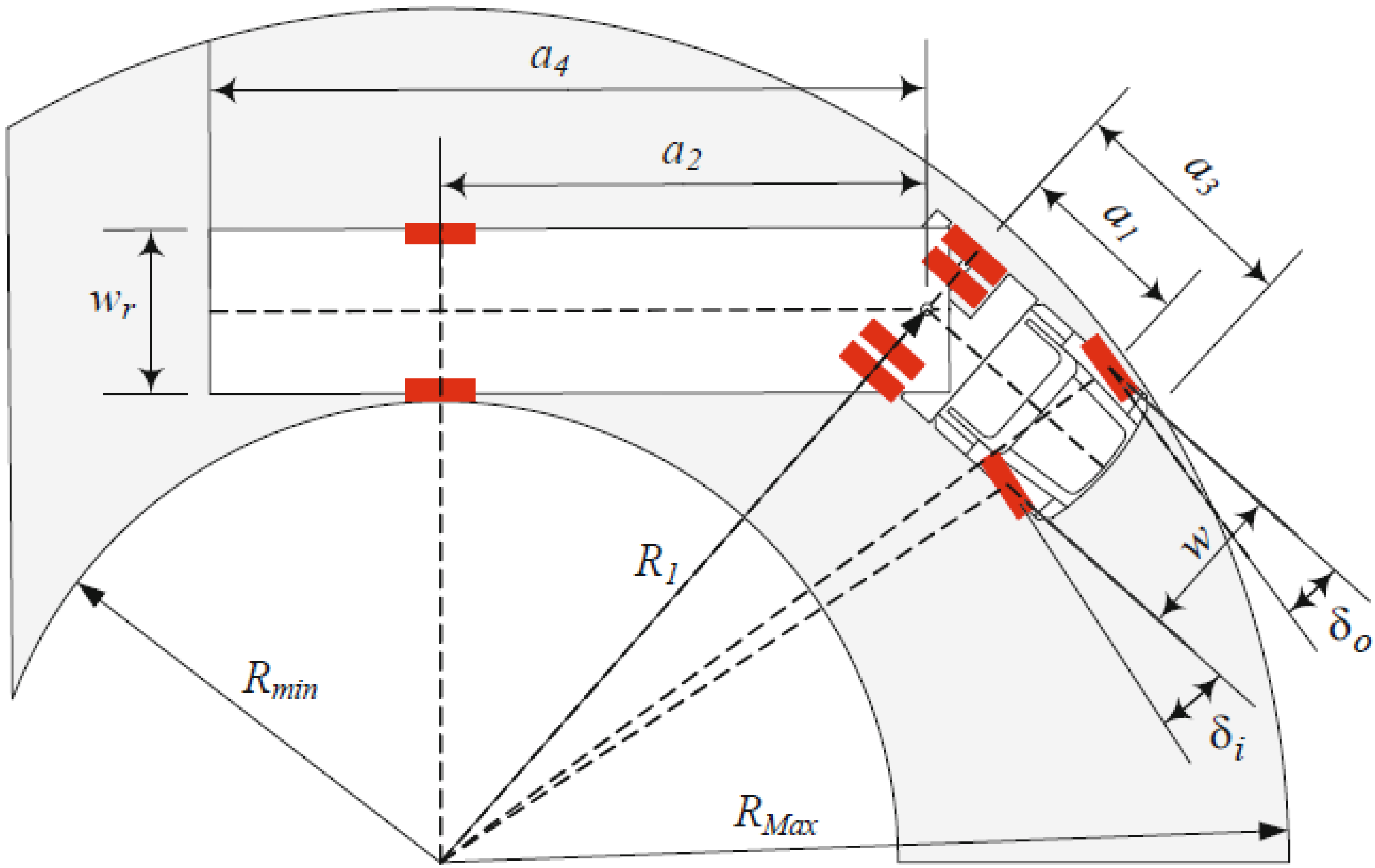
$$= \sqrt{\left(\frac{l}{\tan\delta_o} + w\right)^2 + (l+g)^2} - \frac{l}{\tan\delta_o} + w$$





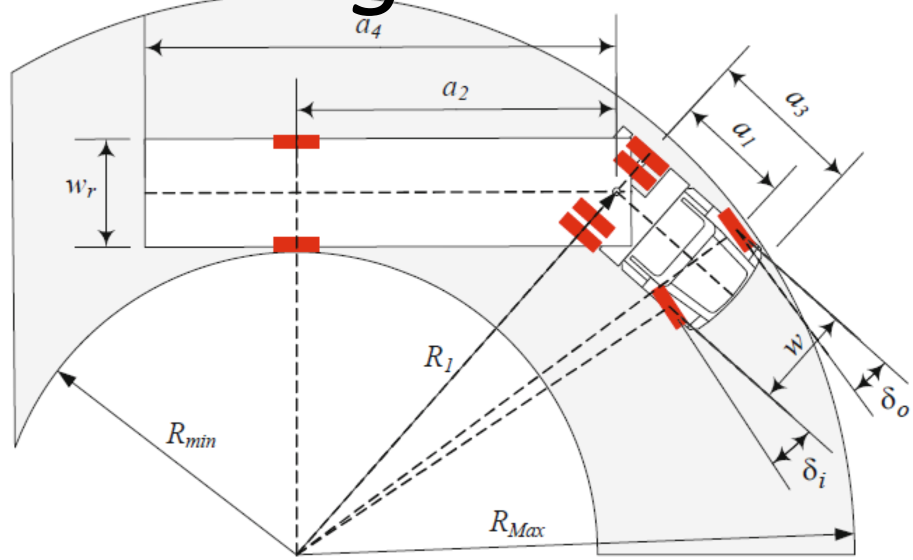
Kinematic *Steering*

A semitrailer in a turn



Kinematic Steering

A semitrailer in a turn



$$\Delta R = R_{Max} - R_{min} = \sqrt{\left(R_1 + \frac{w}{2}\right)^2 + a_3^2} - \sqrt{R_1 - a_2^2} - \frac{w_r}{2}$$

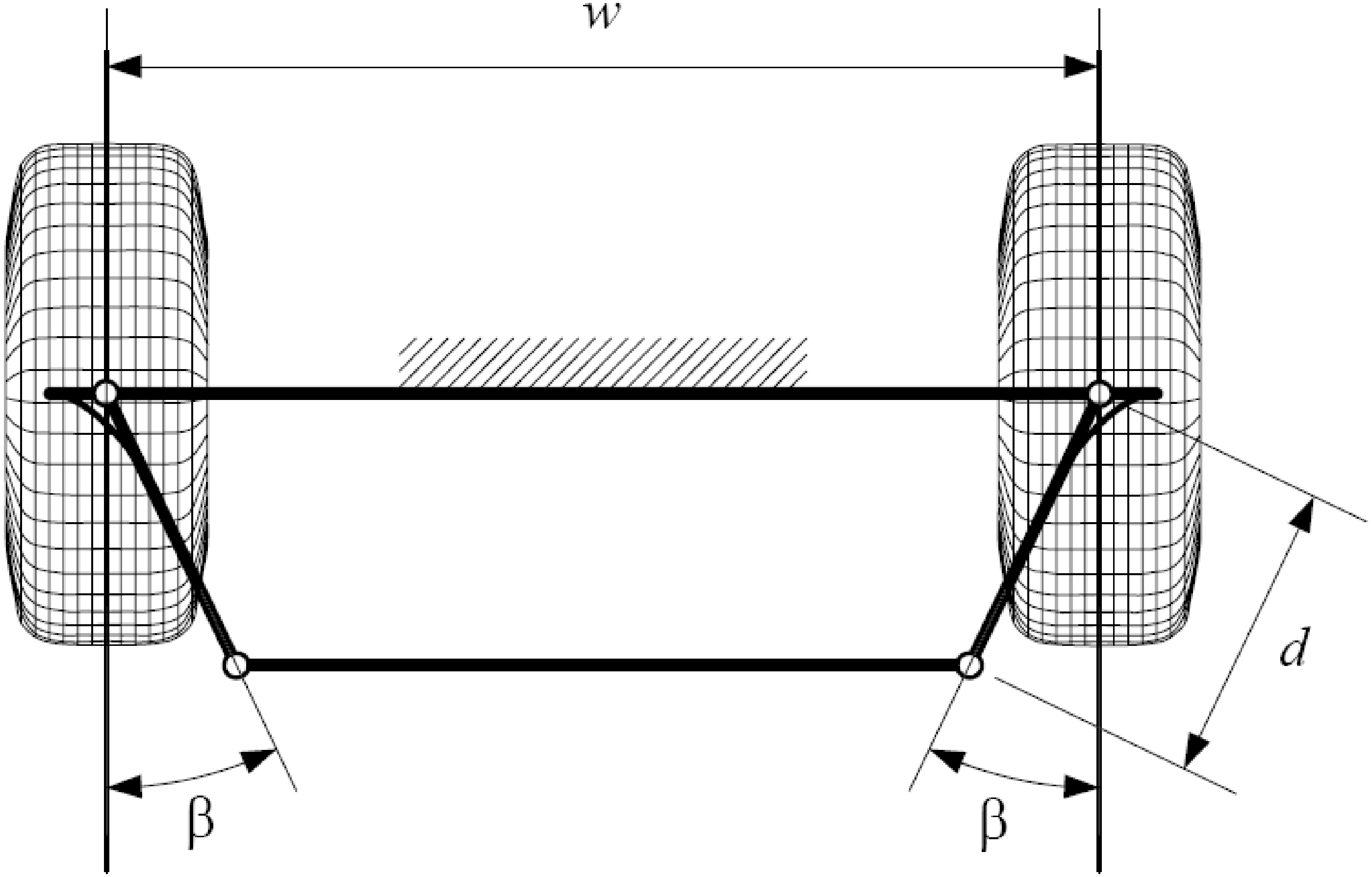
$$R_1 = \frac{1}{2}w + \frac{a_1}{\tan\delta_i} = -\frac{1}{2}w + \frac{a_1}{\tan\delta_o}$$

$$R_{Max} = \sqrt{\left(R_1 + \frac{w}{2}\right)^2 + a_3^2}$$

$$R_{min} = \sqrt{R_1 - a_2^2} - \frac{1}{2}w_r$$

Trapezoidal Steering Mechanism

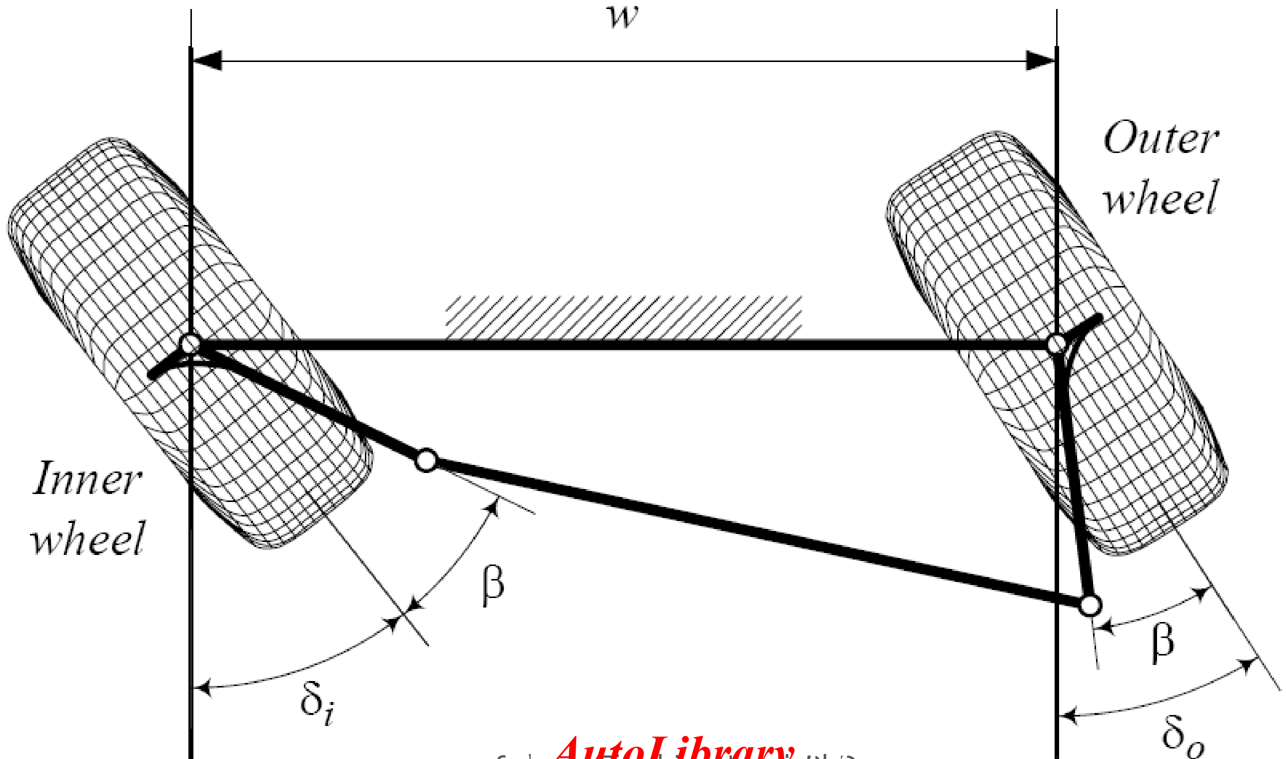
A trapezoidal steering mechanism



Trapezoidal Steering Mechanism

Steered configuration of a trapezoidal steering mechanism

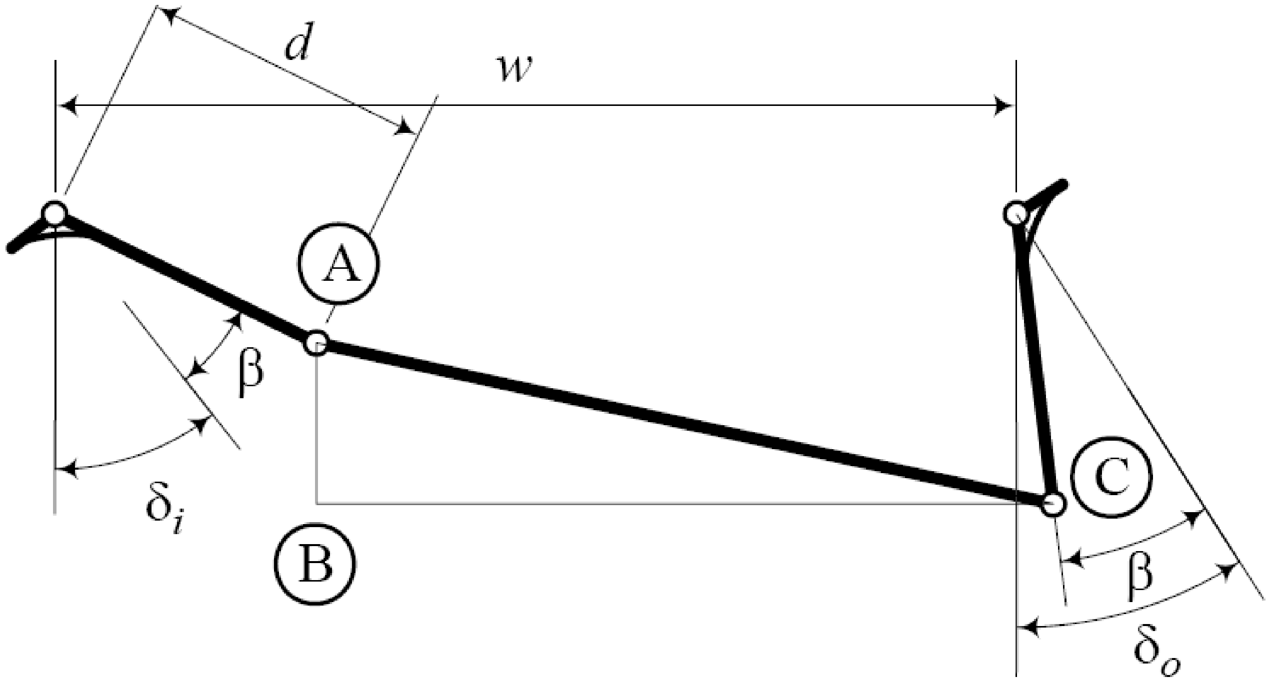
$$\sin(\beta + \delta_i) + \sin(\beta - \delta_o)$$
$$= \frac{w}{d} + \sqrt{\left(\frac{w}{d} - 2\sin\beta\right)^2 - \left(\cos(\beta - \delta_o) - \cos(\beta + \delta_i)\right)^2}$$



Trapezoidal Steering Mechanism

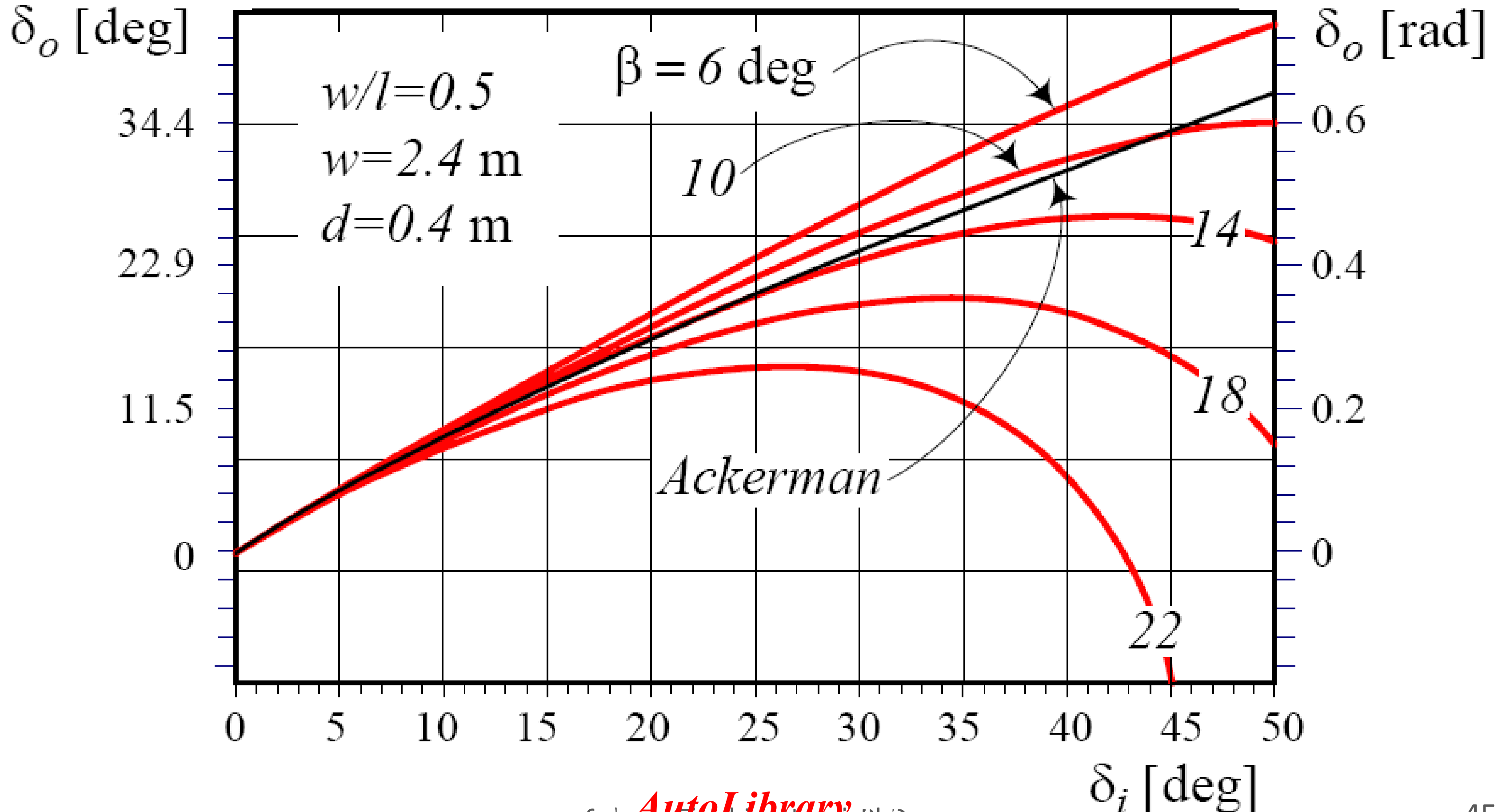
Proof: Using triangle ABC

$$
\begin{aligned}
(w - 2d\sin\beta)^2 \quad = \quad & (w - d\sin(\beta + \delta_i) - d\sin(\beta - \delta_o))^2 \\
& + (d\cos(\beta - \delta_o) - d\cos(\beta + \delta_i))^2
\end{aligned}
$$



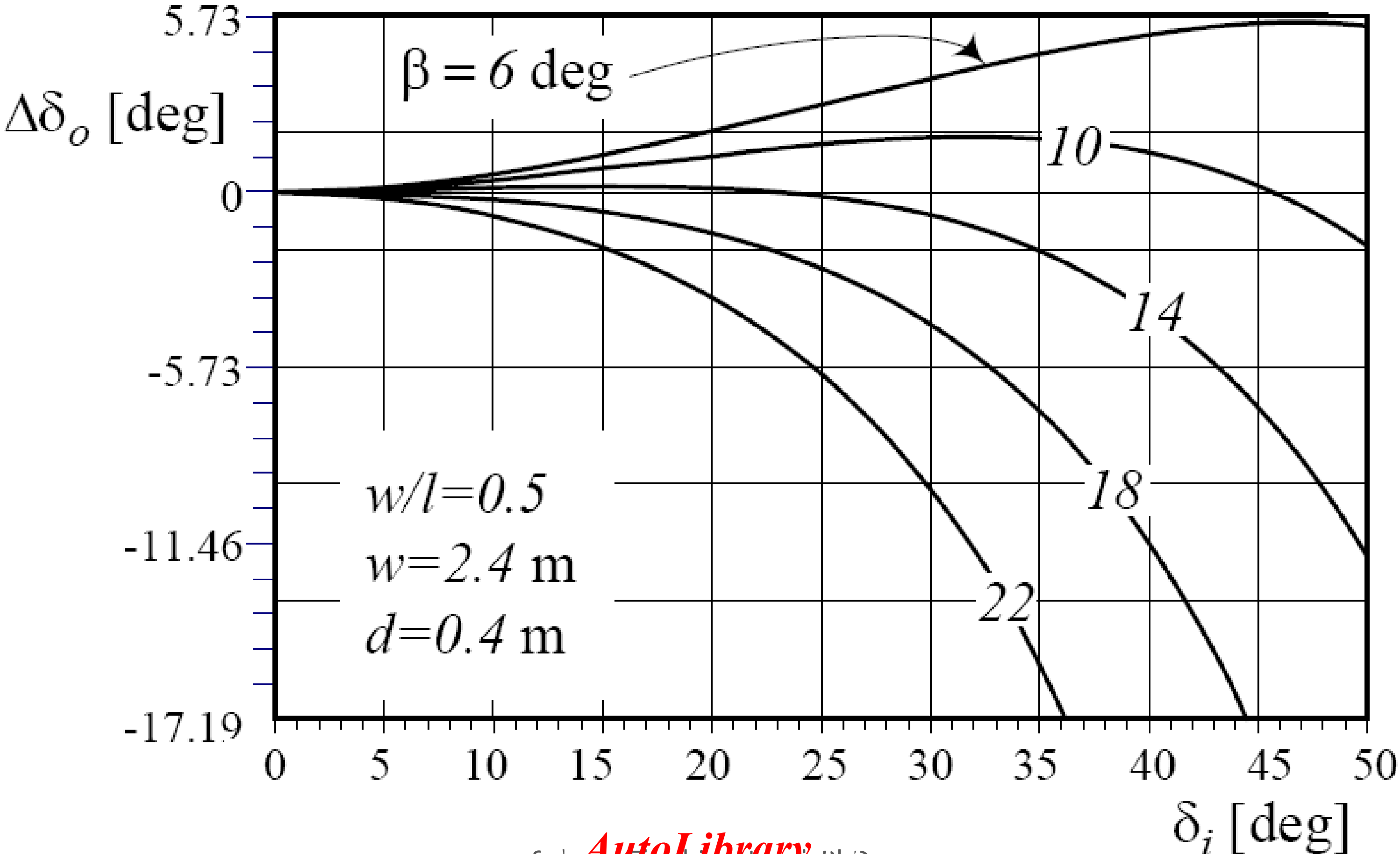
Trapezoidal Steering Mechanism

Behavior of a trapezoidal steering mechanism, compared to the associated Ackerman mechanism.



Trapezoidal Steering Mechanism

The error parameter e = δDo − δAo for a sample trapezoidal steering mechanism



Kinematic *Steering*

Locked rear axle (No differential!)

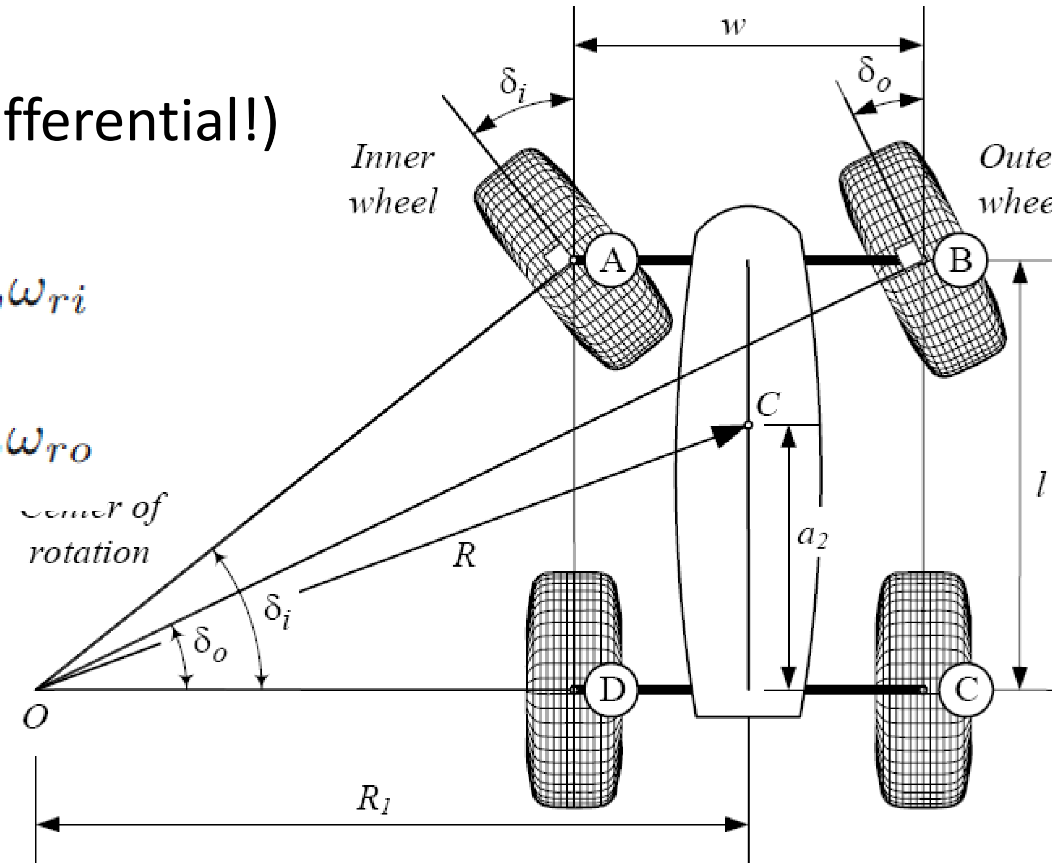
$$v_{ri} = \left(R_1 - \frac{w}{2}\right) r = R_w \omega_{ri}$$

$$v_{ro} = \left(R_1 + \frac{w}{2}\right) r = R_w \omega_{ro}$$

r is yaw rate of the vehicle

$$\omega_{ri} = \omega_{ro} = \omega$$

$$\left(R_1 - \frac{w}{2}\right) \neq \left(R_1 + \frac{w}{2}\right)$$

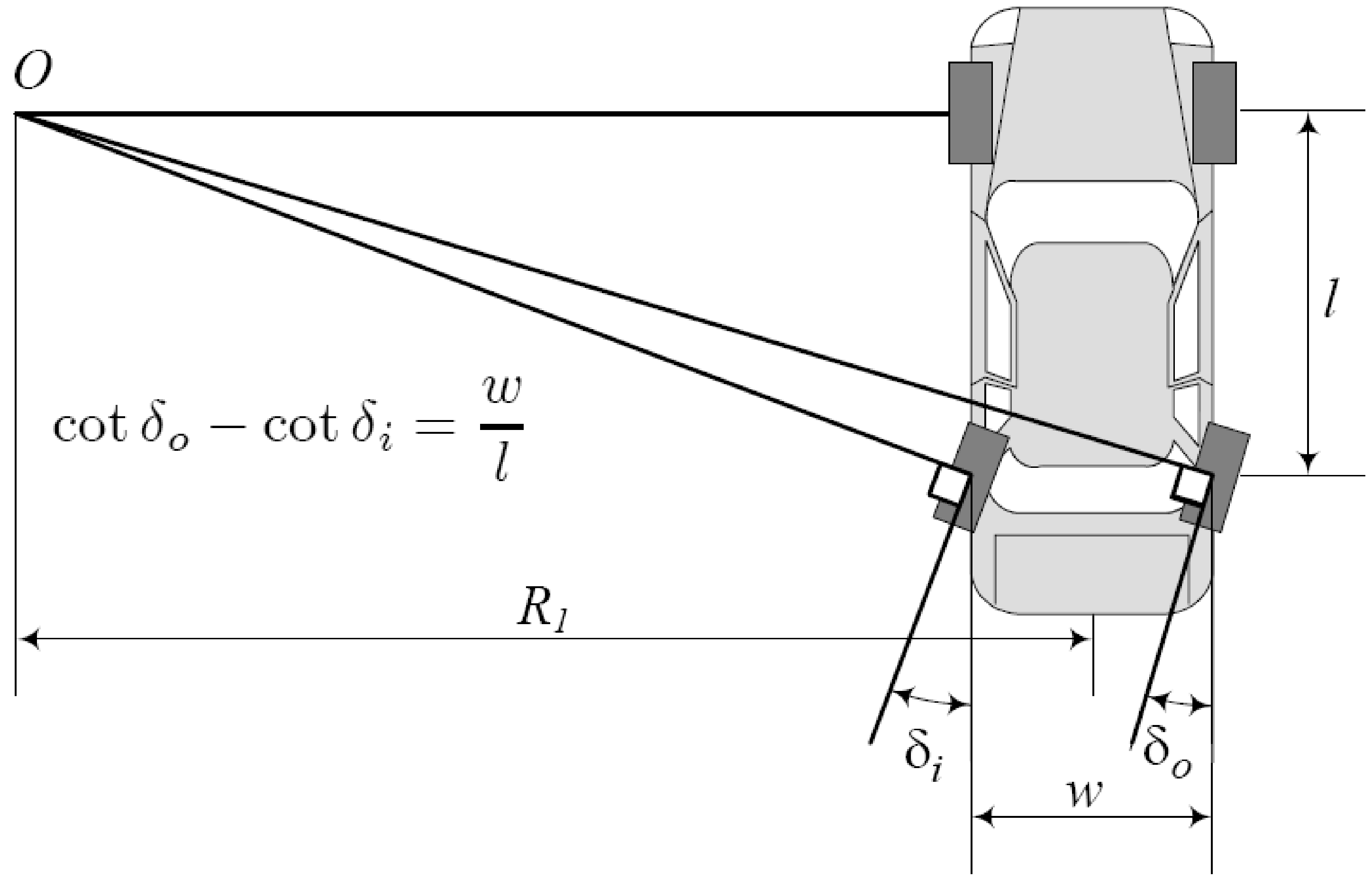


It is impossible to have a locked axle for a nonzero w

Eliminating the differential and using a locked drive axle is an impractical design for street cars. However, it can be an acceptable design for small and light cars moving on dirt or slippery surfaces.

Rear Wheel Steering

A rear-wheel-steering vehicle



$$\cot \delta_o - \cot \delta_i = \frac{w}{l}$$

Steering

Alternative kinematic steer angles equations

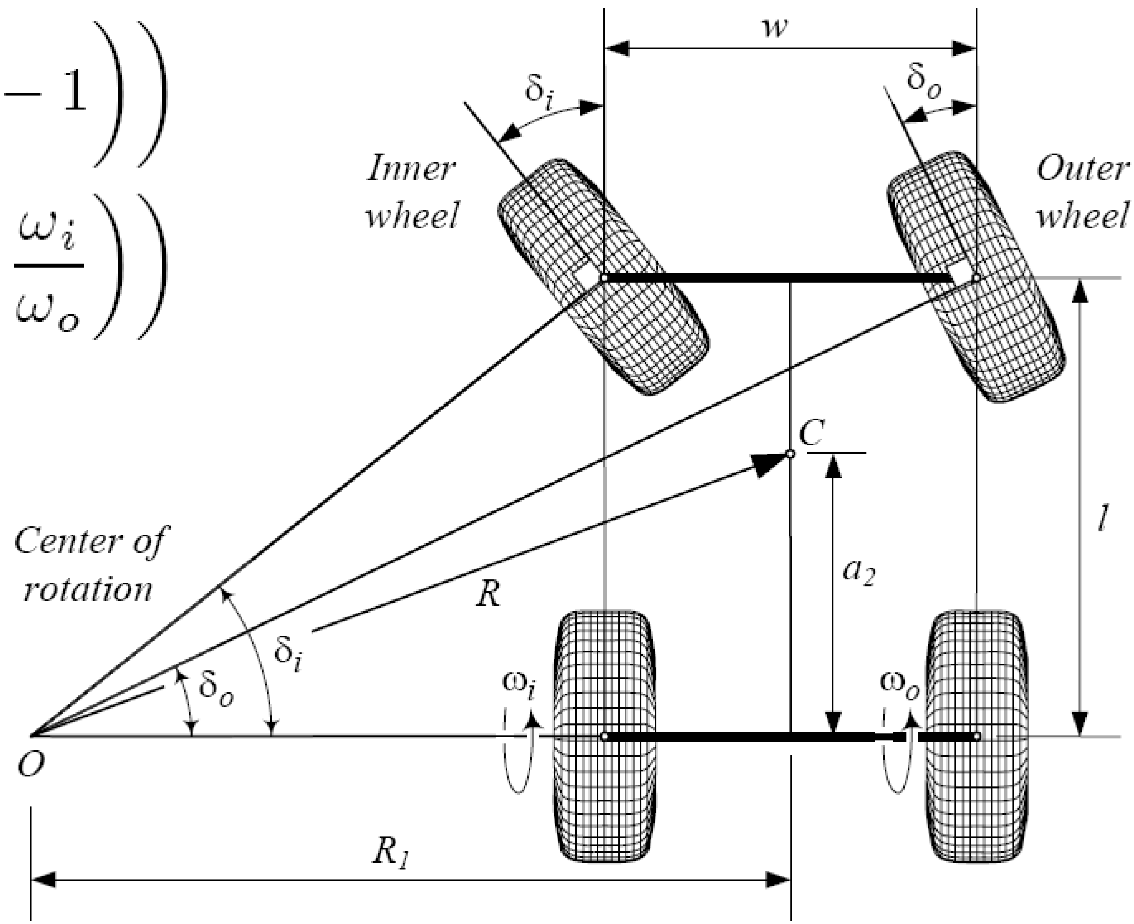
$$\delta_i = \tan^{-1}\left(\frac{l}{w}\left(\frac{\omega_o}{\omega_i} - 1\right)\right)$$

$$\delta_o = \tan^{-1}\left(\frac{l}{w}\left(1 - \frac{\omega_i}{\omega_o}\right)\right)$$

$$\frac{R_w\,\omega_o}{R_1 + \frac{w}{2}} = \frac{R_w\,\omega_i}{R_1 - \frac{w}{2}}$$

Yaw rate:

$$r = \frac{R_w\,\omega_o}{R_1 + \frac{w}{2}} = \frac{R_w\,\omega_i}{R_1 - \frac{w}{2}}$$



Steering

Vehicle with different tracks in the front and in the back

$$\delta_i = \tan^{-1} \frac{2l\,(\omega_o + \omega_i)}{w_f\,(\omega_o - \omega_i) + w_r\,(\omega_o + \omega_i)}$$

$$\delta_o = \tan^{-1} \frac{2l\,(\omega_o - \omega_i)}{w_f\,(\omega_o - \omega_i) + w_r\,(\omega_o + \omega_i)}$$

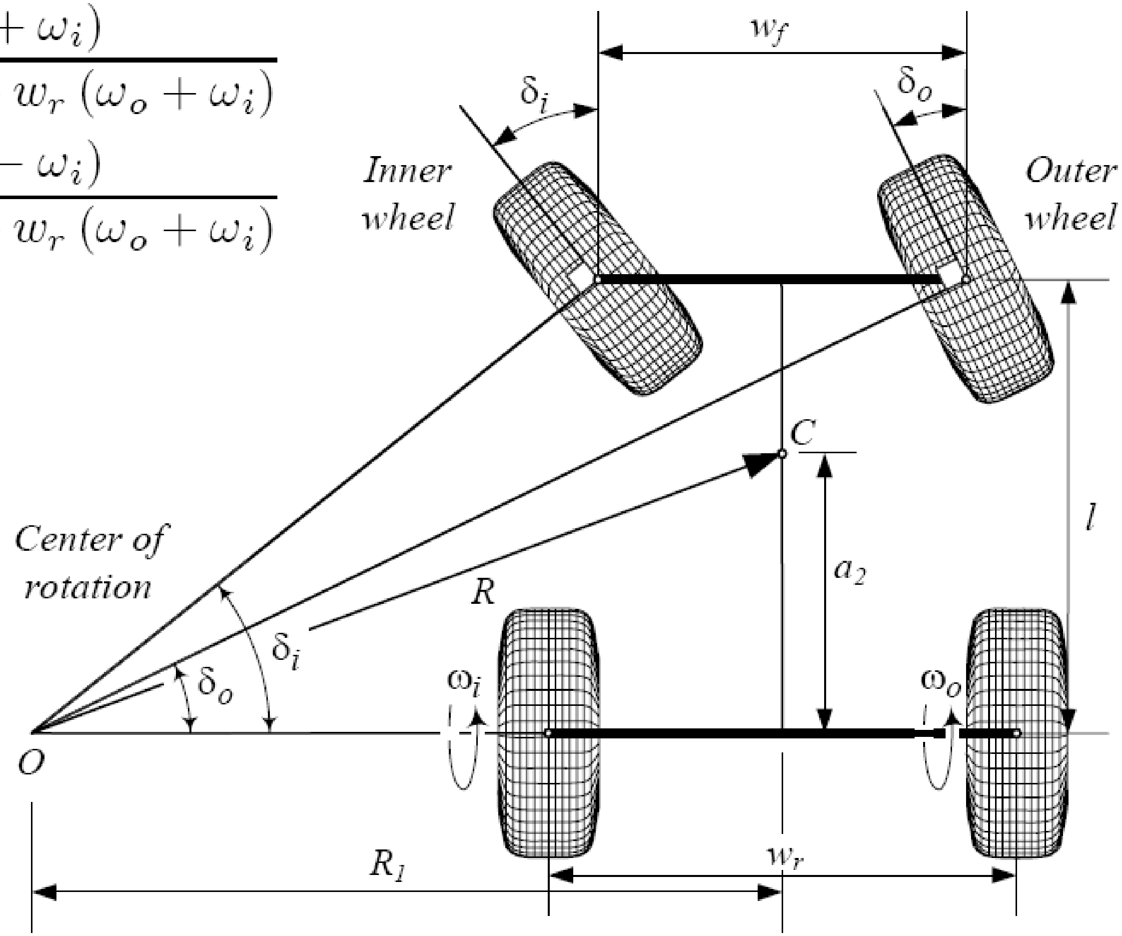
angular velocity of the vehicle

$$r = \frac{R_w\,\omega_o}{R_1 + \frac{w_r}{2}} = \frac{R_w\,\omega_i}{R_1 - \frac{w_r}{2}}$$

$$R_1 = \frac{w_r}{2}\,\frac{\omega_o + \omega_i}{\omega_o - \omega_i}$$

$$\tan\delta_i = \frac{l}{R_1 - \frac{w_f}{2}}$$

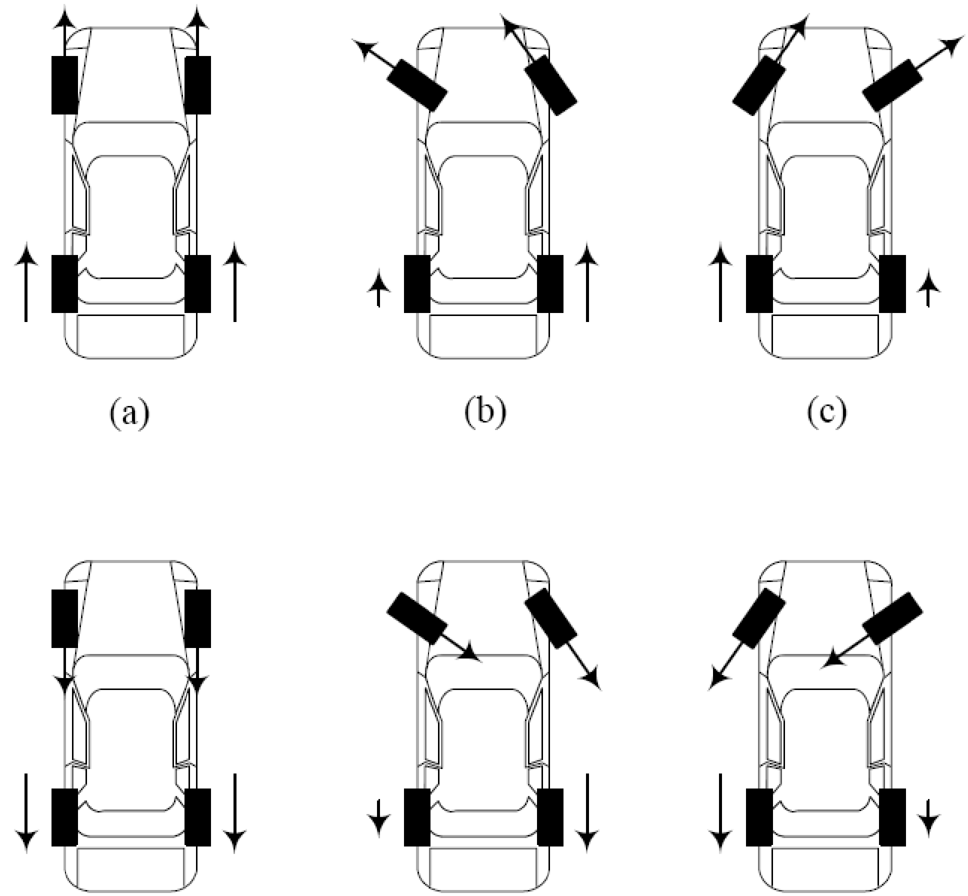
$$\tan\delta_o = \frac{l}{R_1 + \frac{w_f}{2}}$$



A Highly Steerable Vehicle

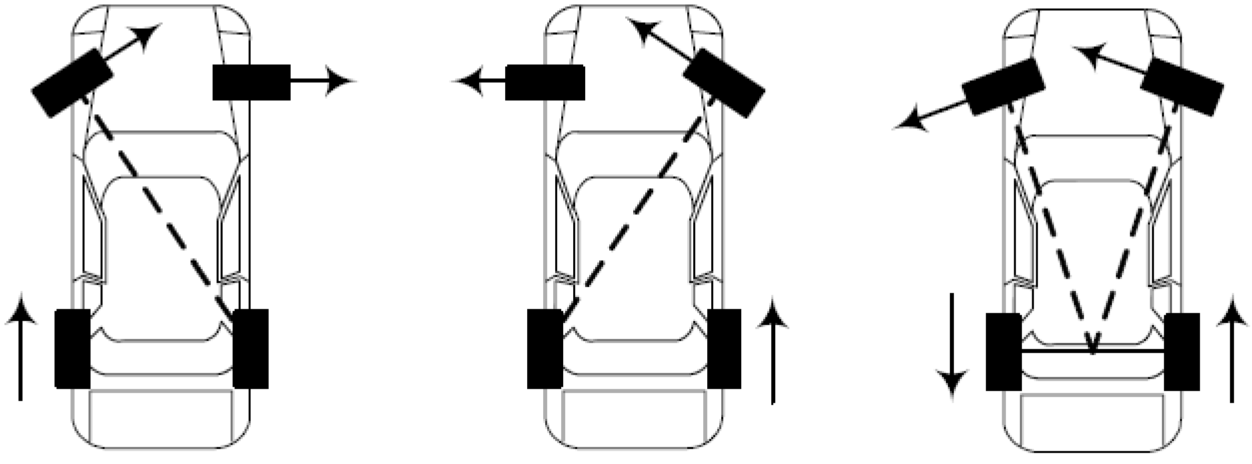
For some special-purpose vehicles, we may design an independent rear-wheel-drive vehicle by attaching each drive wheel to an independently controlled motor to apply any desired angular velocity.

The steerable wheels of such vehicles are usually designed to be able to turn more than 90 deg to the left and right. Such a vehicle is highly maneuverable at low speeds.



(a) (b) (c)

A Highly Steerable Vehicle



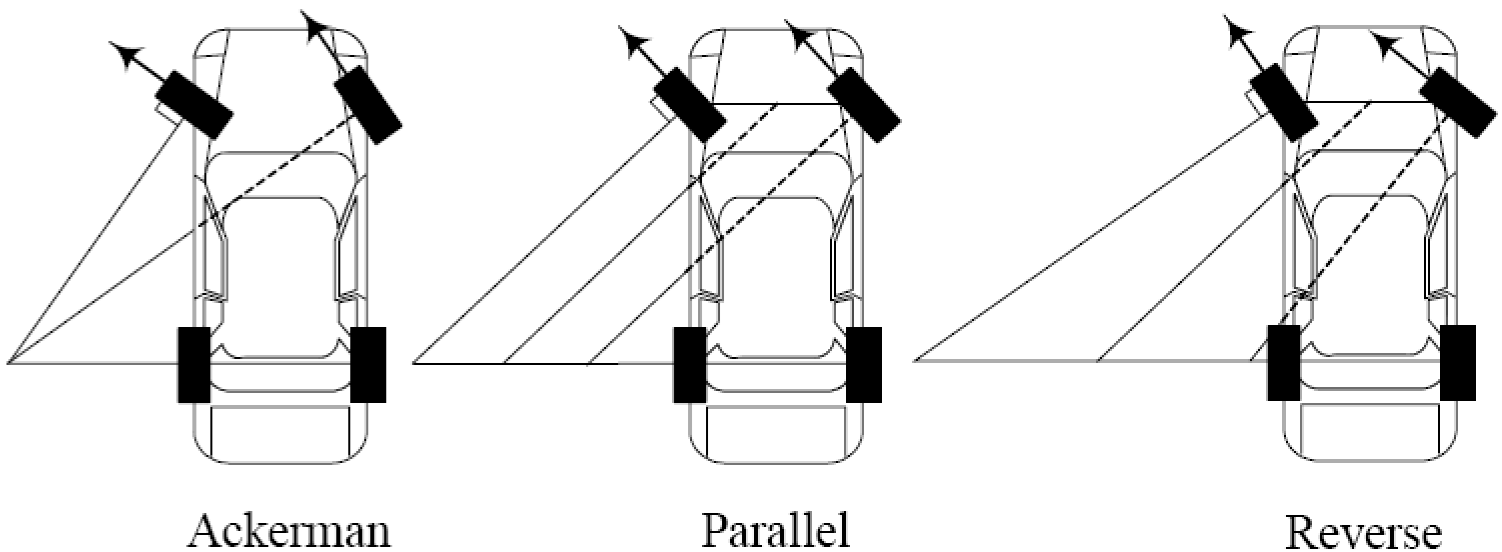
(g) (h) (i)

$$\frac{\omega_o}{\omega_i} = \frac{\delta_o\,(w_f - w_r) - 2l}{\delta_o\,(w_f + w_r) - 2l}$$

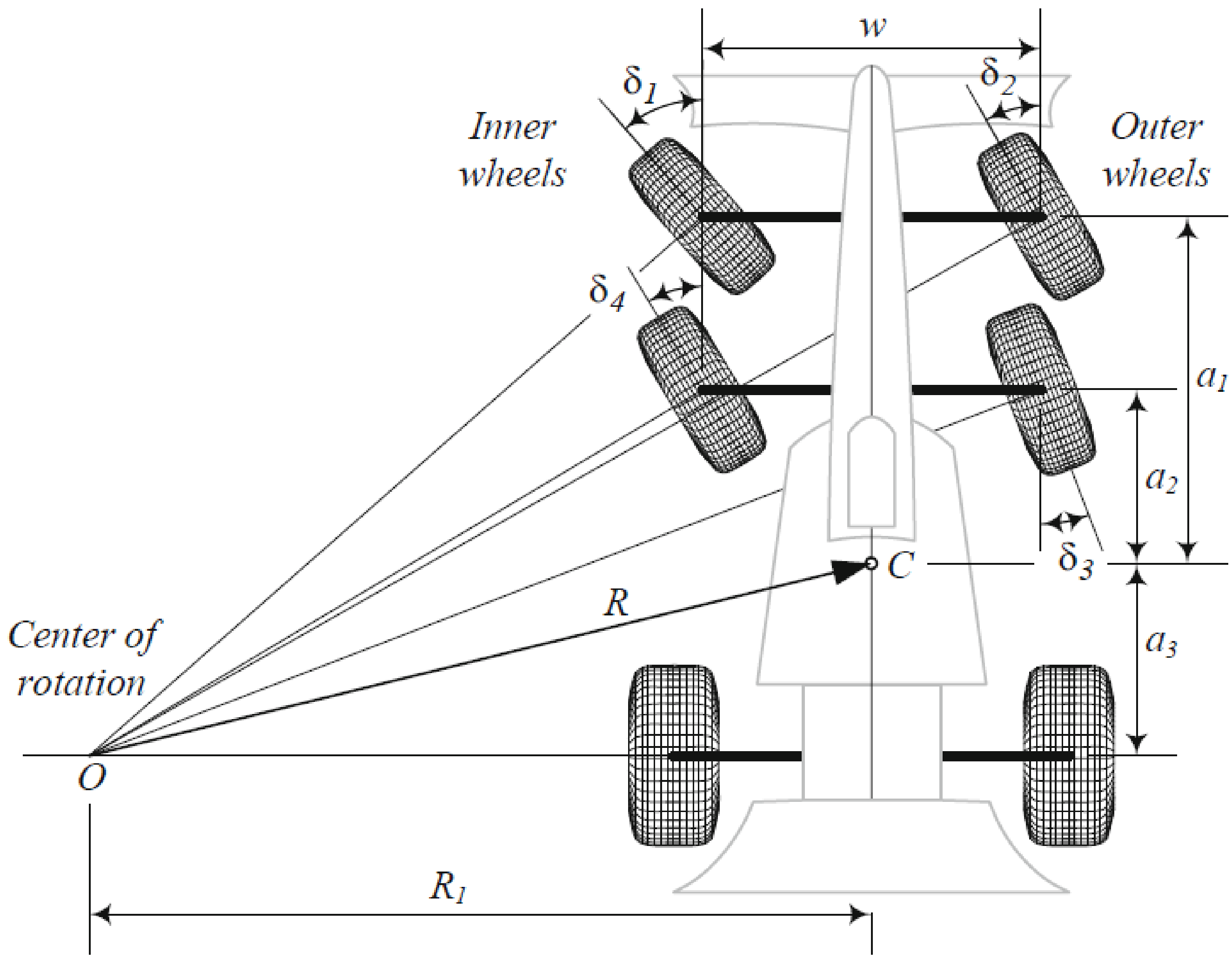
$$\frac{\omega_o}{\omega_i} = \frac{\delta_i\,(w_f + w_r) + 2l}{\delta_i\,(w_f - w_r) + 2l}$$

Steering

Ackerman steering is a correct condition when the turning speed of the vehicle is very slow. When the vehicle turns very fast, significant lateral acceleration is needed and therefore, the wheels operate at high slip angles. The inner front wheel of a kinematic steering vehicle might be at a higher slip angle than required for maximum lateral force. Therefore, the inner wheel of a vehicle in a high speed turn must operate at a lower steer angle than kinematic steering and the outer wheel must operate at a higher steer angle than kinematic steering, reducing the difference between inner and outer wheel steer angles. For race cars, it is common to use parallel or reverse steering.



Vehicles with More Than Two Axles

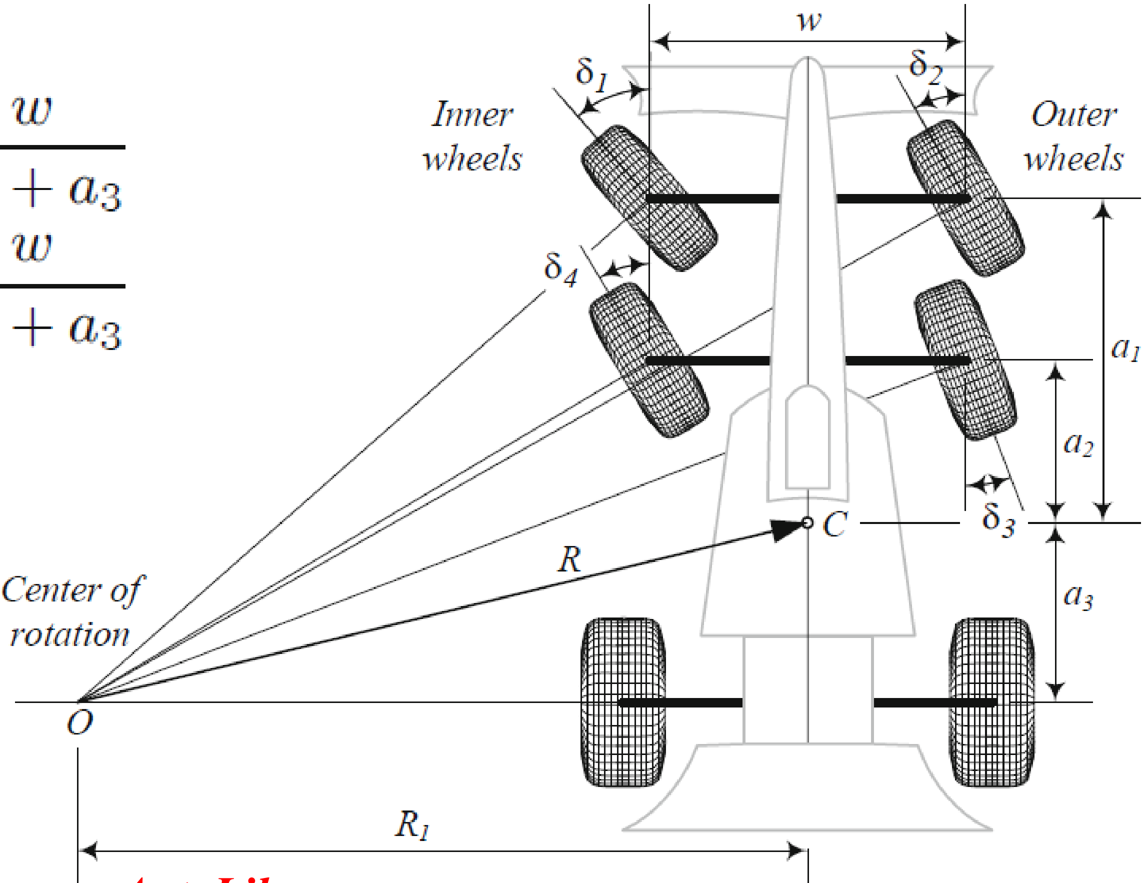


Vehicles with More Than Two Axles

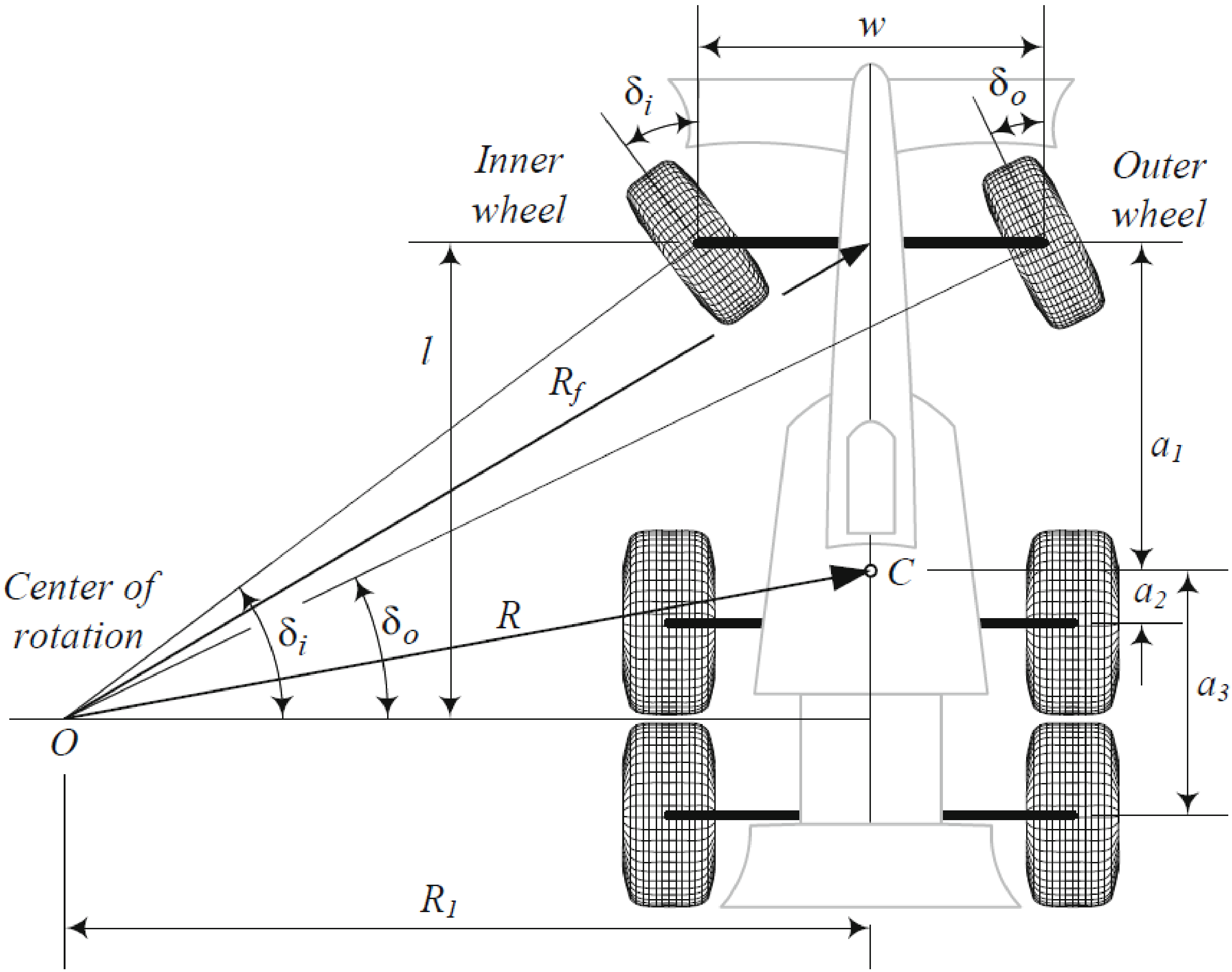
If a vehicle has more than two axles, all the axles, except one, must be steerable to provide slip-free turning at zero velocity. When an n-axle vehicle has only one non-steerable axle, there are n-1 geometric steering conditions.

$$\cot \delta_2 - \cot \delta_1 = \frac{w}{a_1 + a_3}$$

$$\cot \delta_3 - \cot \delta_4 = \frac{w}{a_2 + a_3}$$



A six-wheel vehicle with one steerable axle



A six-wheel vehicle with one steerable axle

Slip-free rotation is impossible for the non-steering wheels.

We design the steering mechanism such that the center of rotation O is on a lateral line, called the midline, between the coupled rear axles.

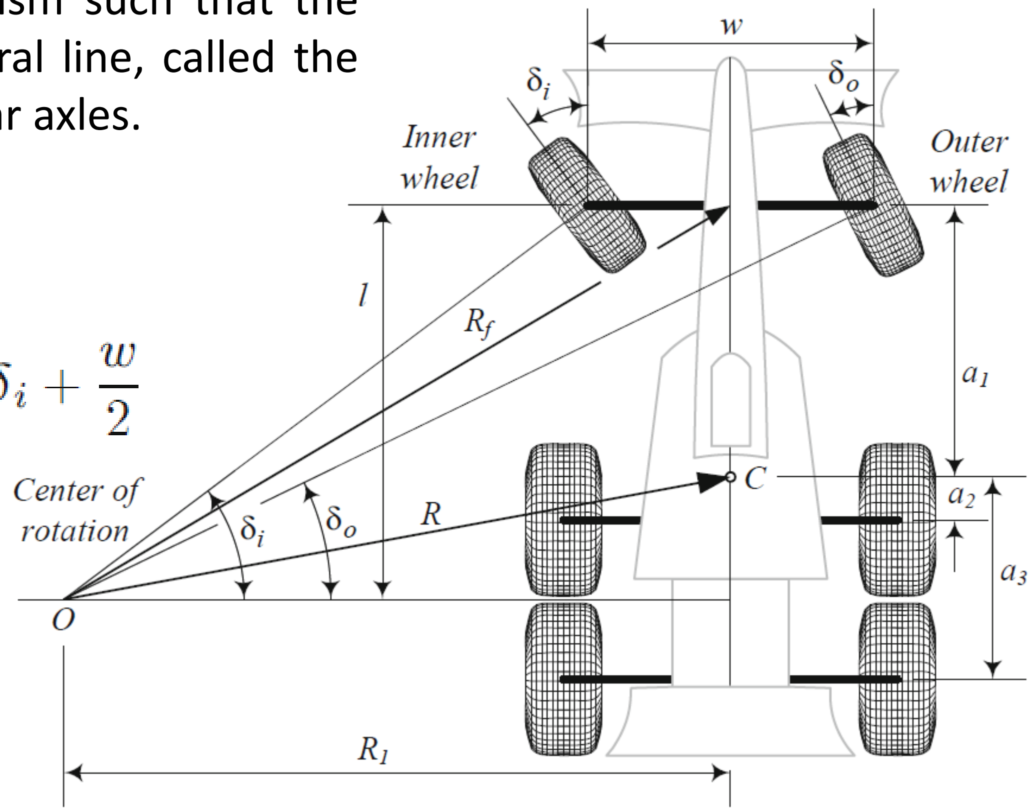
$$l = a_1 + a_2 + \frac{a_3 - a_2}{2}$$

$$R_1 = l \cot \delta_o - \frac{w}{2} = l \cot \delta_i + \frac{w}{2}$$

$$\cot \delta_o - \cot \delta_i = \frac{w}{l}$$

$$R_f = \frac{R_1}{\cos\left(\tan^{-1} \frac{l}{R_1}\right)}$$

$$R = \frac{R_1}{\cos\left(\tan^{-1} \frac{a_3 + a_2}{2R_1}\right)}$$



Vehicle with Trailer

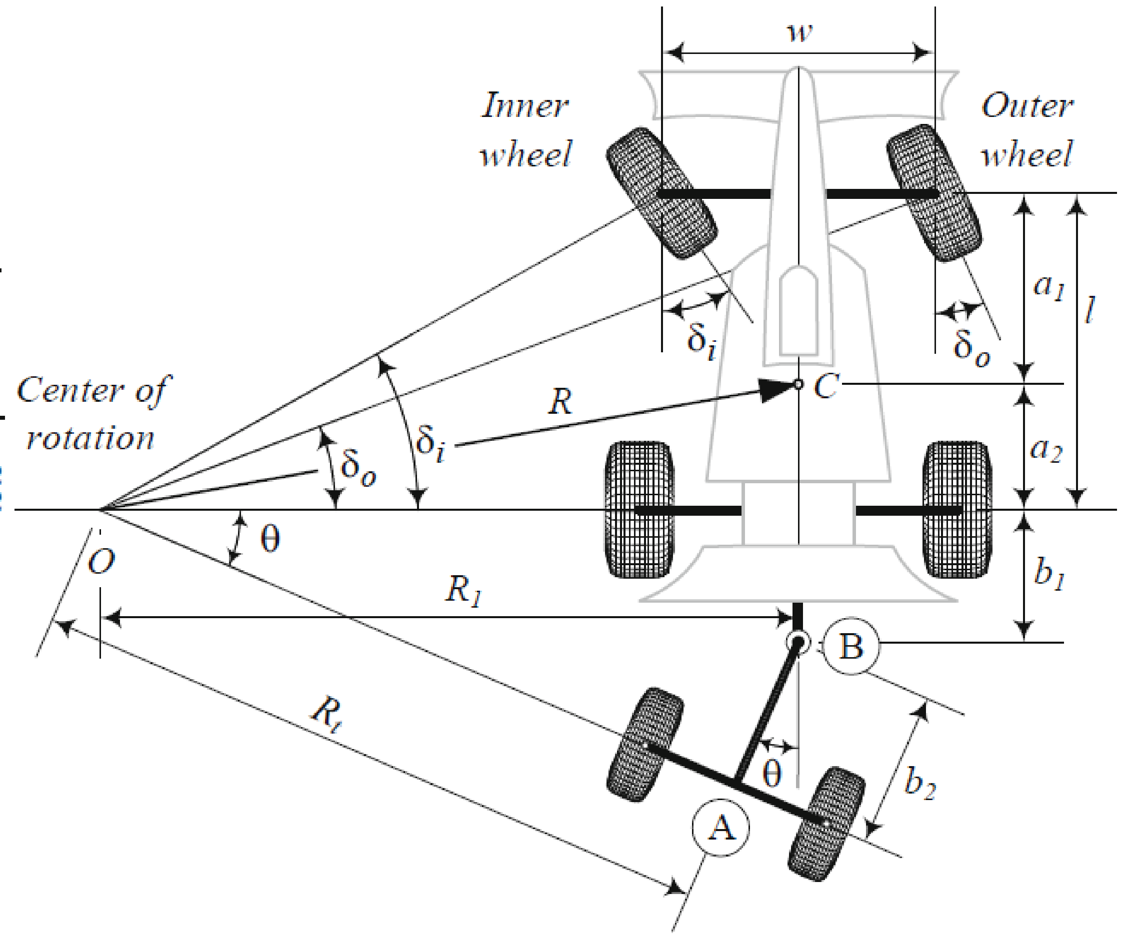
$$R_t = \sqrt{R_1^2 + b_1^2 - b_2^2}$$

$$\overline{OB}^2 = R_t^2 + b_2^2 = R_1^2 + b_1^2$$

$$R_t = \sqrt{\left(l \cot \delta_i + \frac{1}{2}w\right)^2 + b_1^2 - b_2^2}$$

$$R_t = \sqrt{\left(l \cot \delta_o - \frac{1}{2}w\right)^2 + b_1^2 - b_2^2}$$

$$R_t = \sqrt{R^2 - a_2^2 + b_1^2 - b_2^2}$$



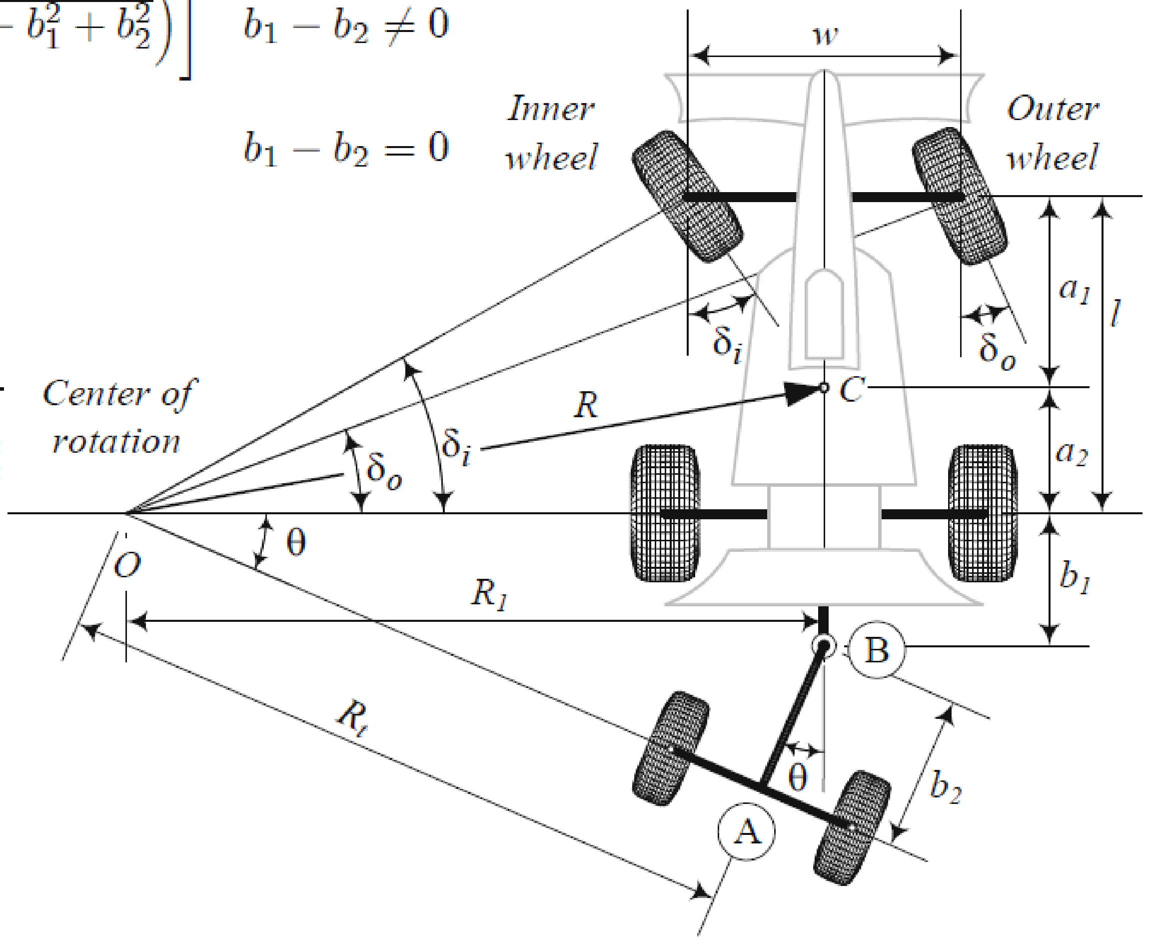
$$R_t \sin\theta = b_1 + b_2 \cos\theta$$

Vehicle with Trailer

$$\theta = \begin{cases} 2\tan^{-1}\left[\frac{1}{b_1 - b_2}\left(R_t \mp \sqrt{R_t^2 - b_1^2 + b_2^2}\right)\right] & b_1 - b_2 \neq 0 \\ 2\tan^{-1}\frac{1}{2R_t}(b_1 + b_2) & b_1 - b_2 = 0 \end{cases}$$

$$R_t = \sqrt{\left(l\cot\delta_i + \frac{1}{2}w\right)^2 + b_1^2 - b_2^2}$$

$$R_t = \sqrt{\left(l\cot\delta_o - \frac{1}{2}w\right)^2 + b_1^2 - b_2^2}$$



Vehicle with Trailer (Space requirement)

$$R_{min} = R_t - \frac{1}{2}w_t = \sqrt{\left(l \cot \delta_i + \frac{1}{2}w\right)^2 + b_1^2 - b_2^2} - \frac{1}{2}w_t$$

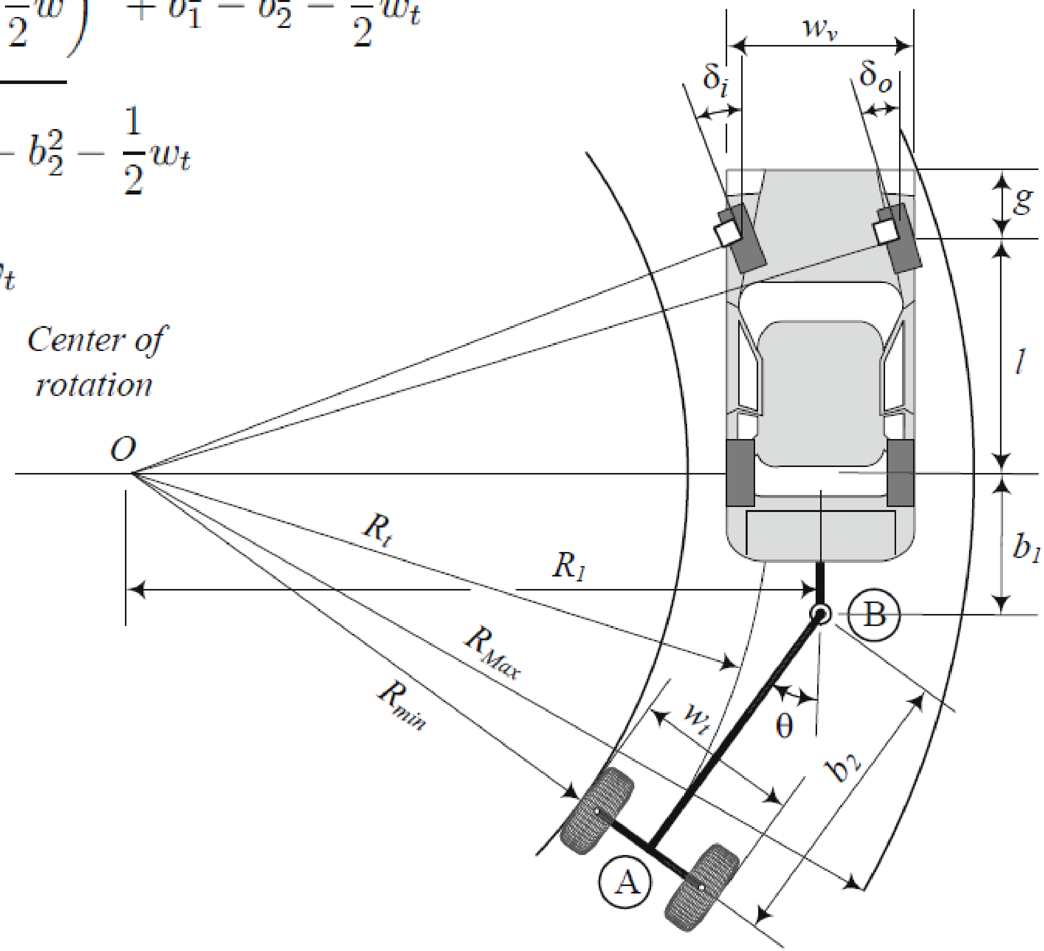
$$= \sqrt{\left(l \cot \delta_o - \frac{1}{2}w\right)^2 + b_1^2 - b_2^2} - \frac{1}{2}w_t$$

$$= \sqrt{R^2 - a_2^2 + b_1^2 - b_2^2} - \frac{1}{2}w_t$$

$$R_{Max} = \sqrt{\left(R_1 + \frac{w_v}{2}\right)^2 + (l+g)^2}$$

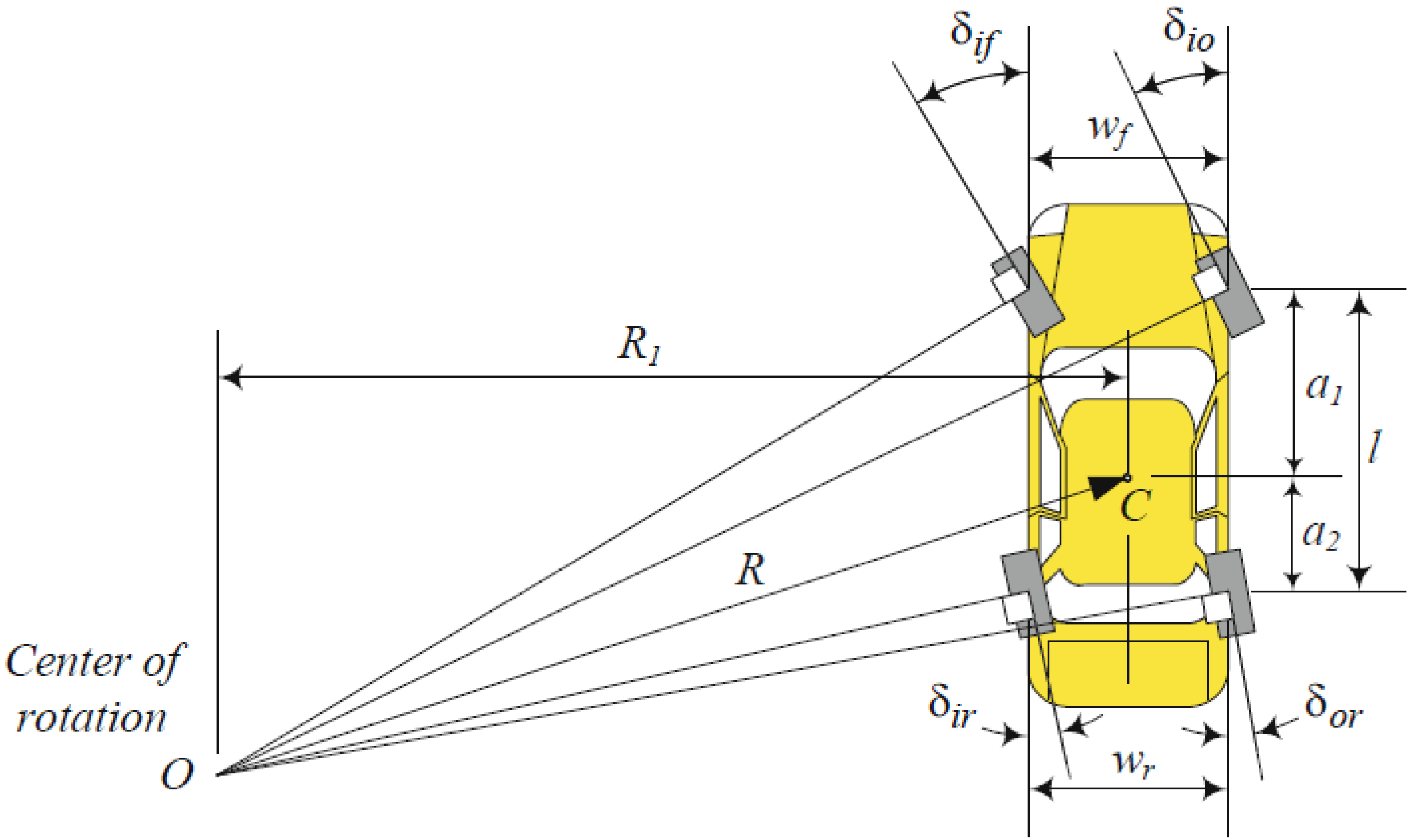
$$R_1 = \sqrt{\left(R_{min} + \frac{w_t}{2}\right)^2 + b_2^2 - b_1^2}$$

$$\triangle R = R_{Max} - R_{min}$$



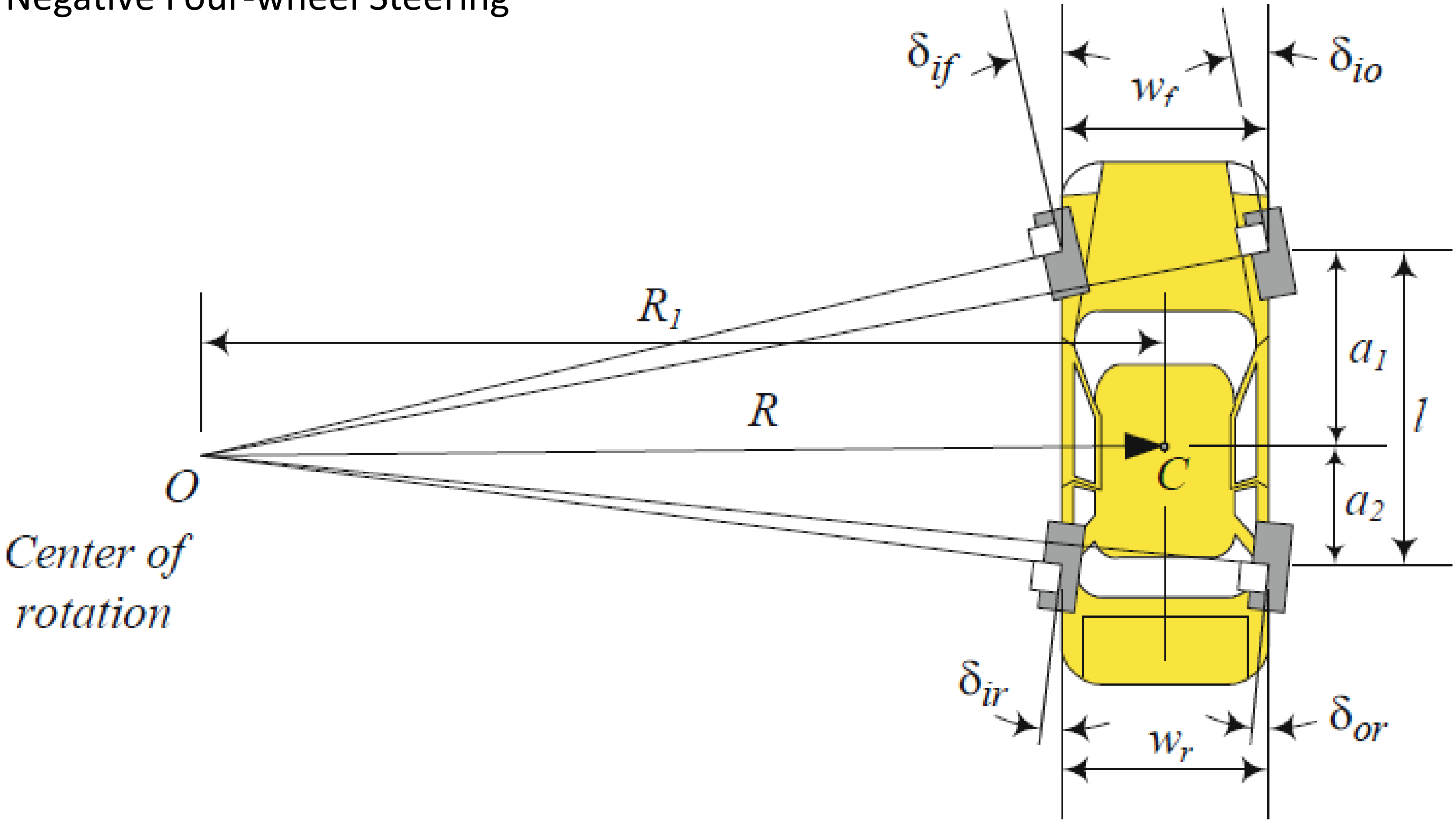
Four-wheel Steering (4WS)

Positive Four-wheel Steering



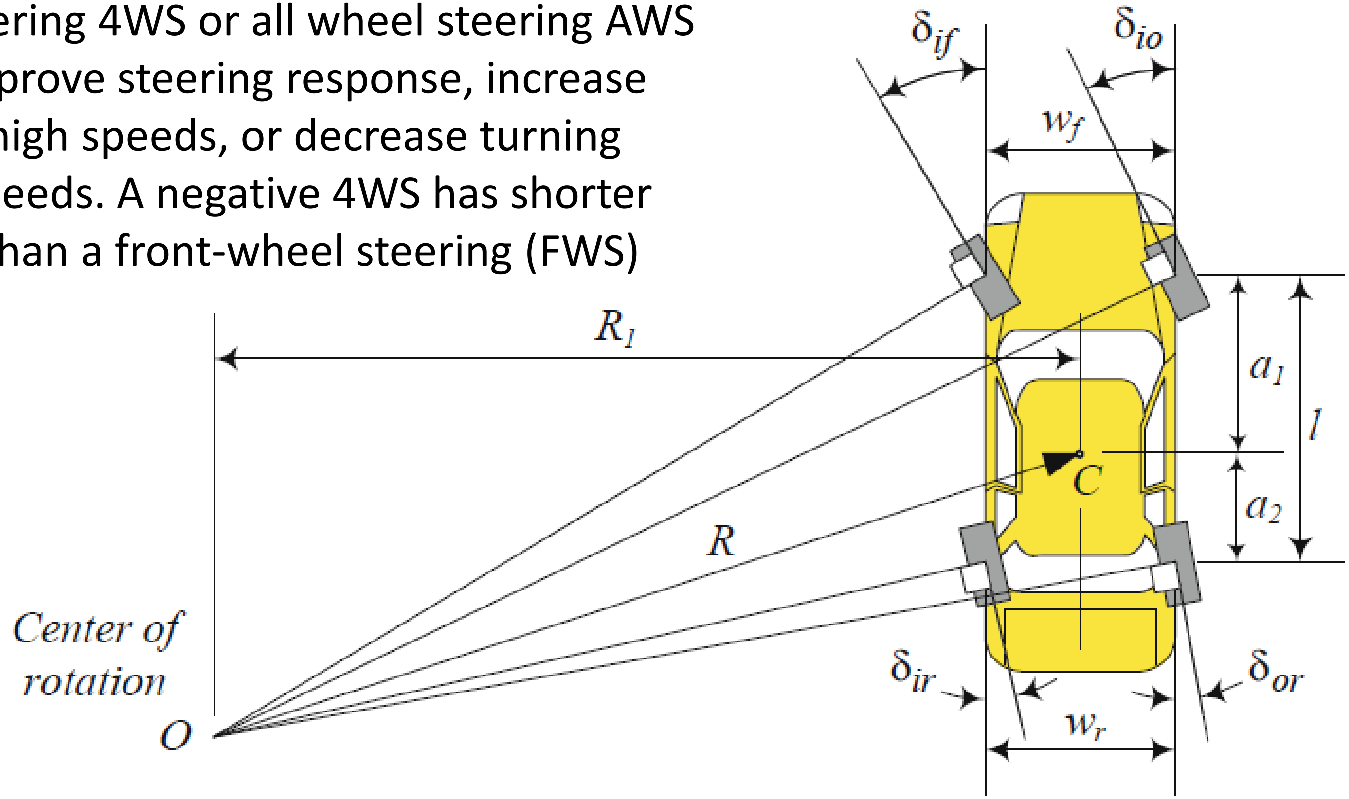
Four-wheel Steering (4WS)

Negative Four-wheel Steering



Four-wheel Steering (4WS)

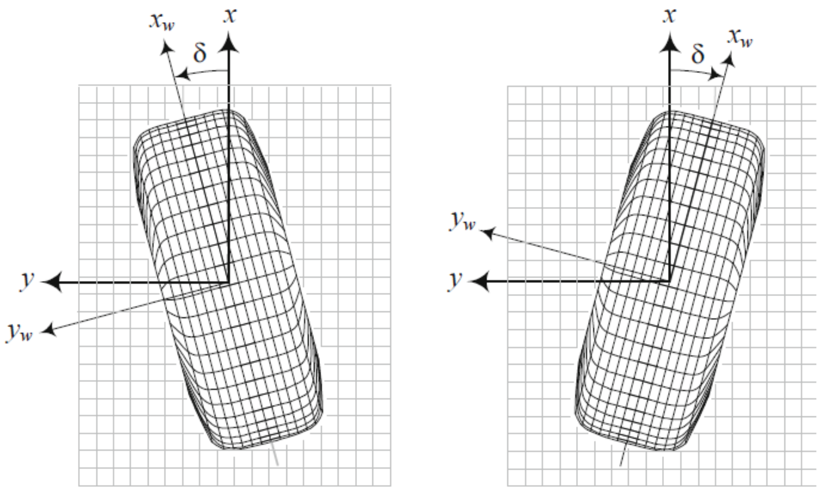
Four-wheel steering 4WS or all wheel steering AWS is applied to improve steering response, increase the stability at high speeds, or decrease turning radius at low speeds. A negative 4WS has shorter turning radius than a front-wheel steering (FWS) vehicle.



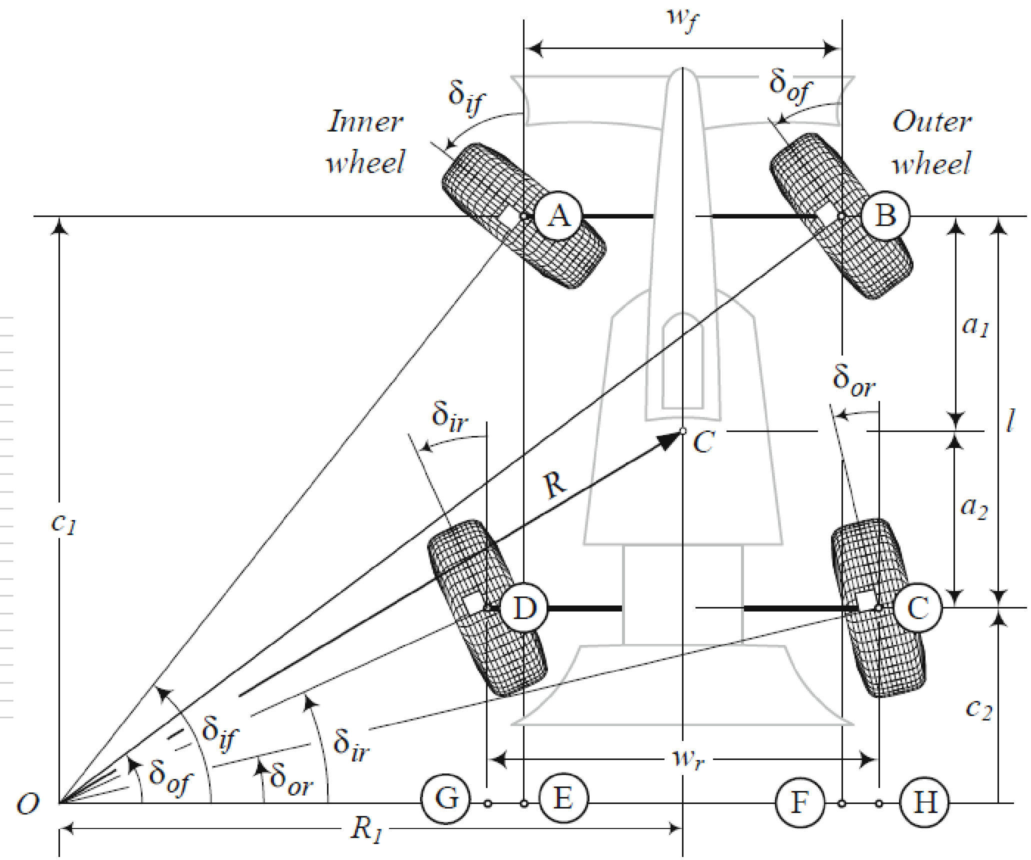
Kinematic Condition in 4WS Vehicle

$$\cot\delta_{of} - \cot\delta_{if} = \frac{w_f}{l} - \frac{w_r}{l}\frac{\cot\delta_{of} - \cot\delta_{if}}{\cot\delta_{or} - \cot\delta_{ir}}$$

Four-wheel Steering (4WS)



Positive steer angle *Negative steer angle*



More general equation

$$\cot \delta_{fr} - \cot \delta_{fl} = \frac{w_f}{l} - \frac{w_r}{l} \frac{\cot \delta_{fr} - \cot \delta_{fl}}{\cot \delta_{rr} - \cot \delta_{rl}}$$

Four-wheel Steering (4WS)

Positive 4WS (Same Steer)

$$\tan\delta_{if} = \frac{c_1}{R_1 - \frac{w_f}{2}} \qquad \tan\delta_{of} = \frac{c_1}{R_1 + \frac{w_f}{2}}$$

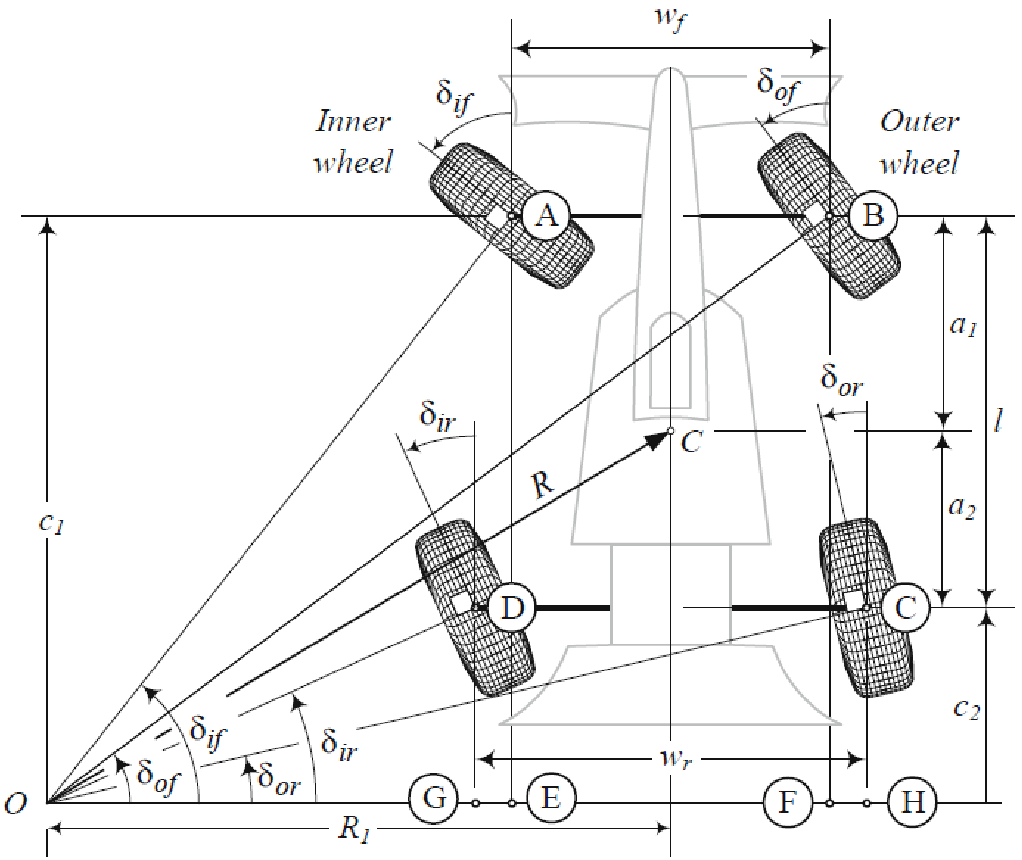
$$R_1 = \frac{1}{2}w_f + \frac{c_1}{\tan\delta_{if}} = -\frac{1}{2}w_f + \frac{c_1}{\tan\delta_{of}}$$

$$\cot\delta_{of} - \cot\delta_{if} = \frac{w_f}{c_1}$$

$$\tan\delta_{ir} = \frac{c_2}{R_1 - \frac{w_r}{2}} \qquad \tan\delta_{or} = \frac{c_2}{R_1 + \frac{w_r}{2}}$$

$$R_1 = \frac{1}{2}w_r + \frac{c_2}{\tan\delta_{ir}} = -\frac{1}{2}w_r + \frac{c_2}{\tan\delta_{or}}$$

$$\cot\delta_{or} - \cot\delta_{ir} = \frac{w_r}{c_2}$$



$$c_1 - c_2 = l$$

$$\frac{w_f}{\cot\delta_{of} - \cot\delta_{if}} - \frac{w_r}{\cot\delta_{or} - \cot\delta_{ir}} = l$$

Four-wheel Steering (4WS)

Negative 4WS (Counter Steer)

$$\tan\delta_{if} = \frac{c_1}{R_1 - \frac{w_f}{2}} \qquad \tan\delta_{of} = \frac{c_1}{R_1 + \frac{w_f}{2}}$$

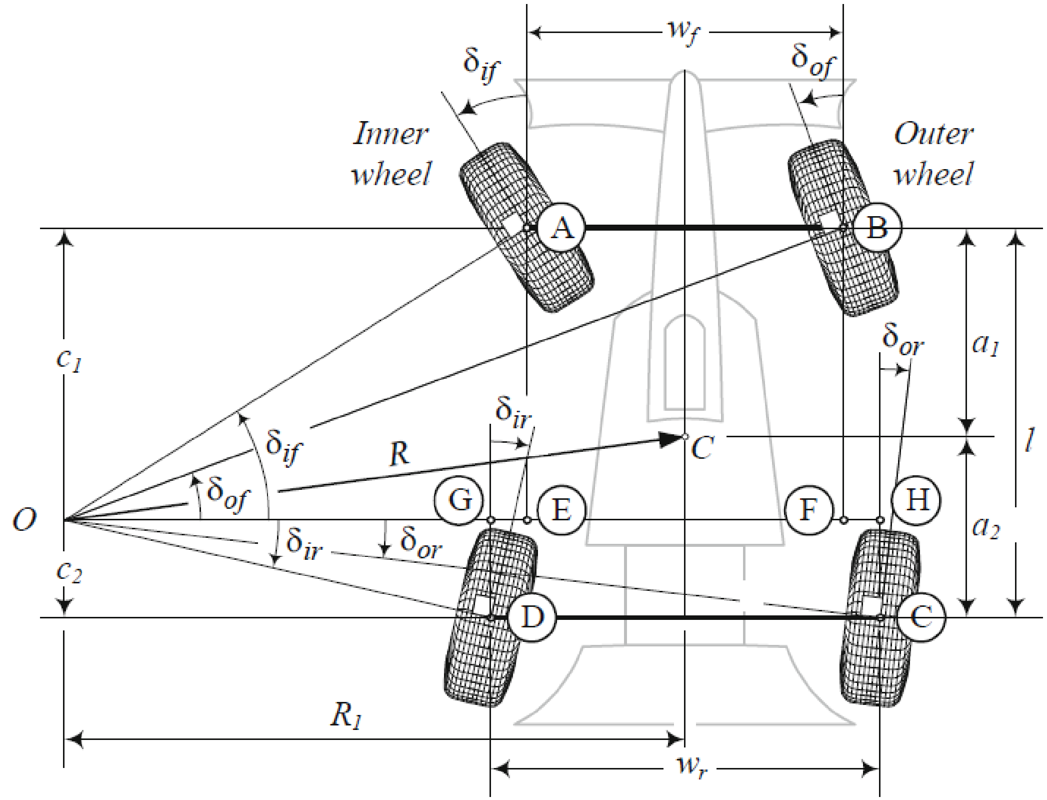
$$R_1 = \frac{1}{2}w_f + \frac{c_1}{\tan\delta_{if}} = -\frac{1}{2}w_f + \frac{c_1}{\tan\delta_{of}}$$

$$\cot\delta_{of} - \cot\delta_{if} = \frac{w_f}{c_1}$$

$$-\tan\delta_{ir} = \frac{-c_2}{R_1 - \frac{w_r}{2}} \qquad -\tan\delta_{or} = \frac{-c_2}{R_1 + \frac{w_r}{2}}$$

$$R_1 = \frac{1}{2}w_f + \frac{c_1}{\tan\delta_{if}} = -\frac{1}{2}w_f + \frac{c_1}{\tan\delta_{of}}$$

$$\cot\delta_{or} - \cot\delta_{ir} = \frac{w_r}{c_2}$$

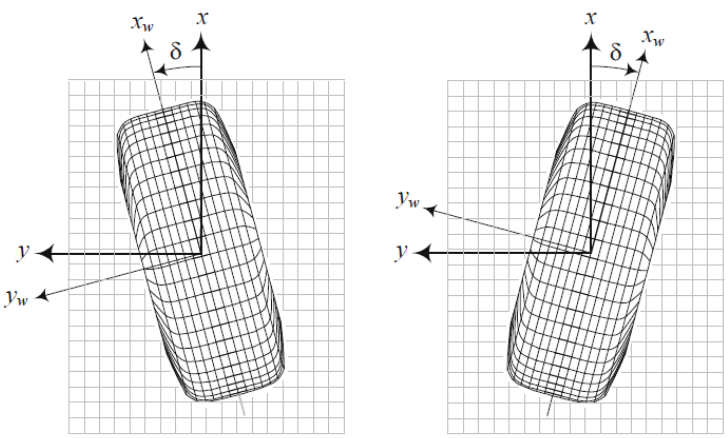


$$c_1 - c_2 = l$$

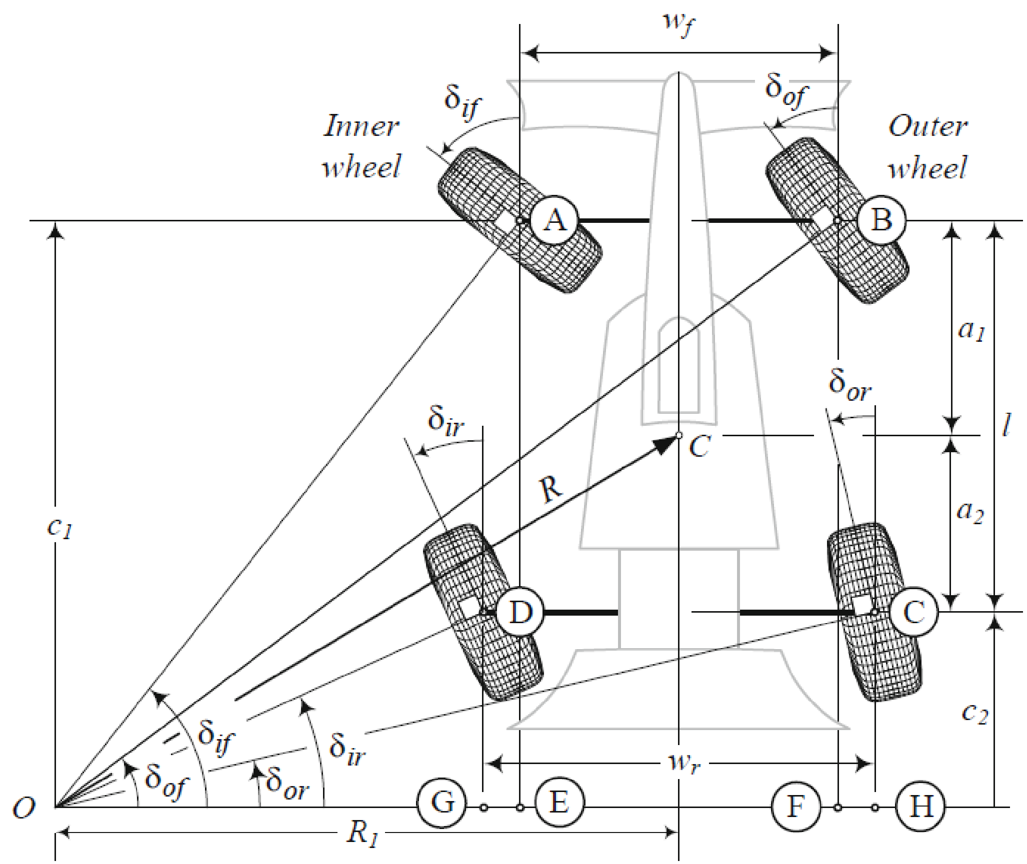
$$\frac{w_f}{\cot\delta_{of} - \cot\delta_{if}} - \frac{w_r}{\cot\delta_{or} - \cot\delta_{ir}} = l$$

Four-wheel Steering (4WS)

More general equation



Positive steer angle   *Negative steer angle*



$$c_1 = \frac{w_f}{\cot\delta_{fr} - \cot\delta_{fl}}$$

$$c_2 = \frac{w_r}{\cot\delta_{rr} - \cot\delta_{rl}}$$

$$\cot\delta_{fr} - \cot\delta_{fl} = \frac{w_f}{l} - \frac{w_r}{l}\,\frac{\cot\delta_{fr} - \cot\delta_{fl}}{\cot\delta_{rr} - \cot\delta_{rl}}$$

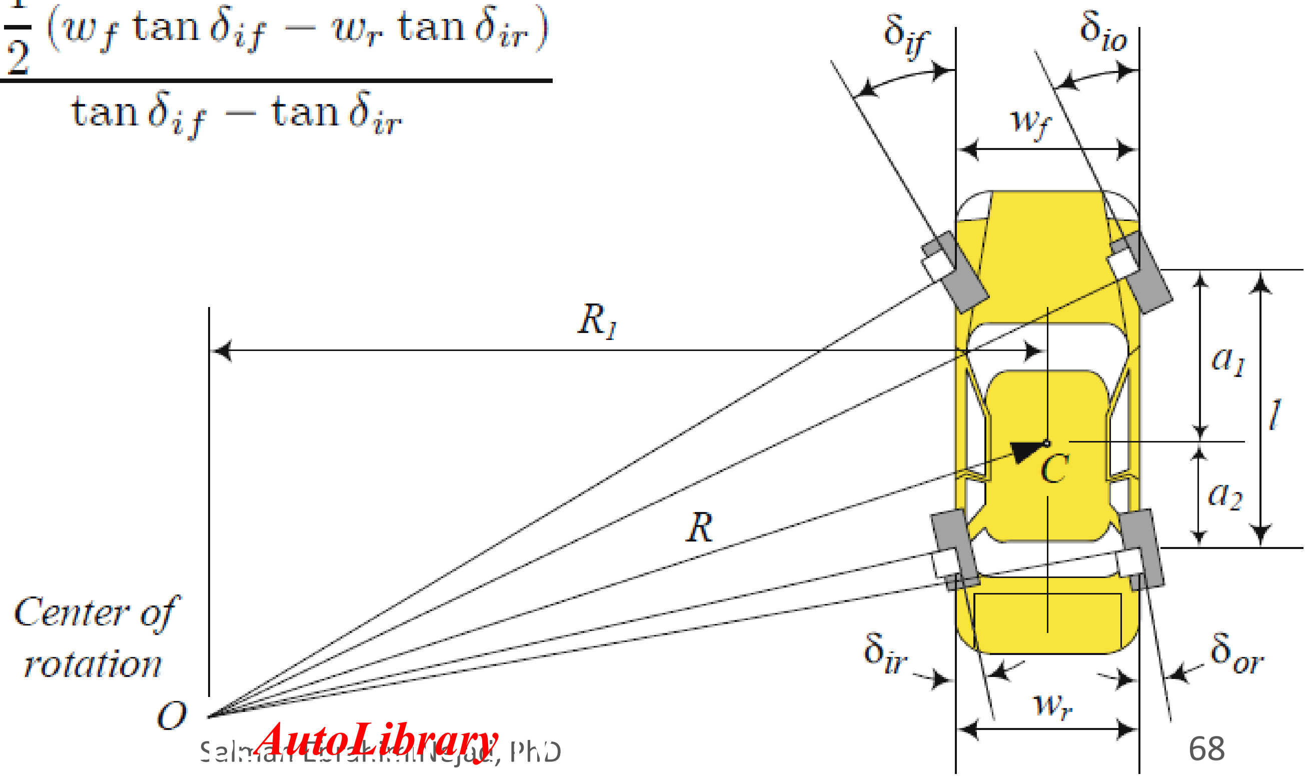
Position of the turning center

In the vehicle body coordinate frame

For a 4WS vehicle:

$$x_O = -a_2 - c_2 = -a_2 - \frac{w_r}{\cot\delta_{or} - \cot\delta_{ir}}$$

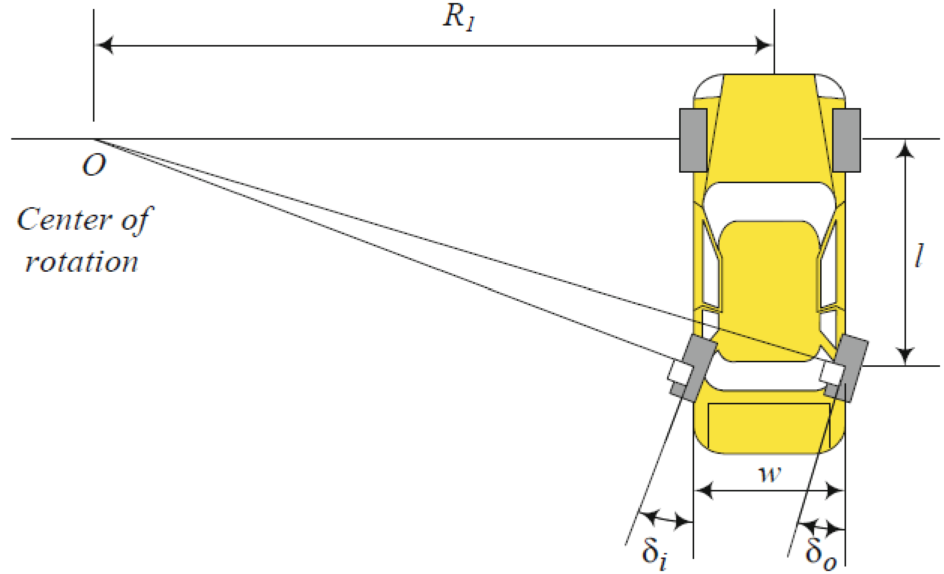
$$y_O = R_1 = \frac{l + \frac{1}{2}\left(w_f \tan\delta_{if} - w_r \tan\delta_{ir}\right)}{\tan\delta_{if} - \tan\delta_{ir}}$$



Position of the turning center

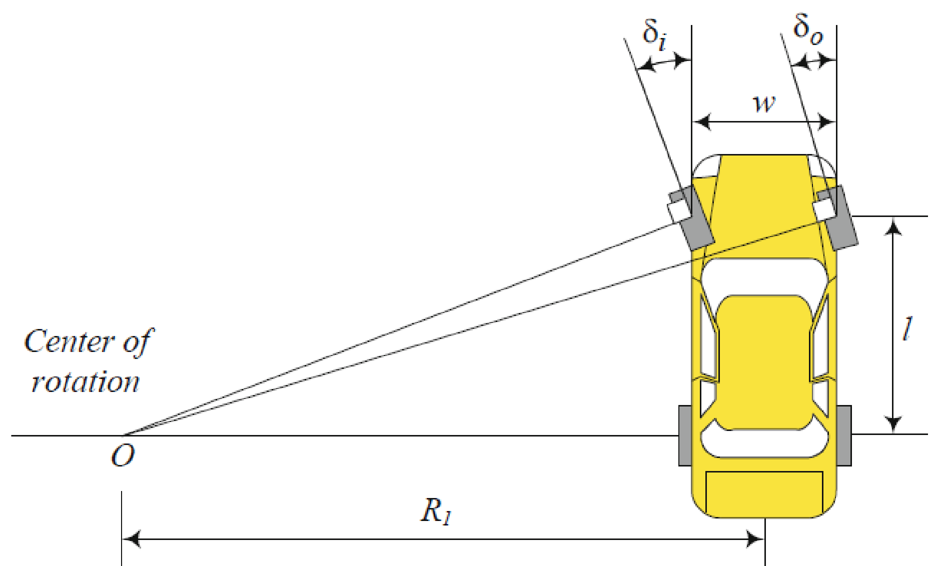
RWS (Rear Wheel Steering) vehicle:

$$x_O = a_1 \qquad y_O = \frac{1}{2}w_r + \frac{l}{\tan\delta_{ir}}$$



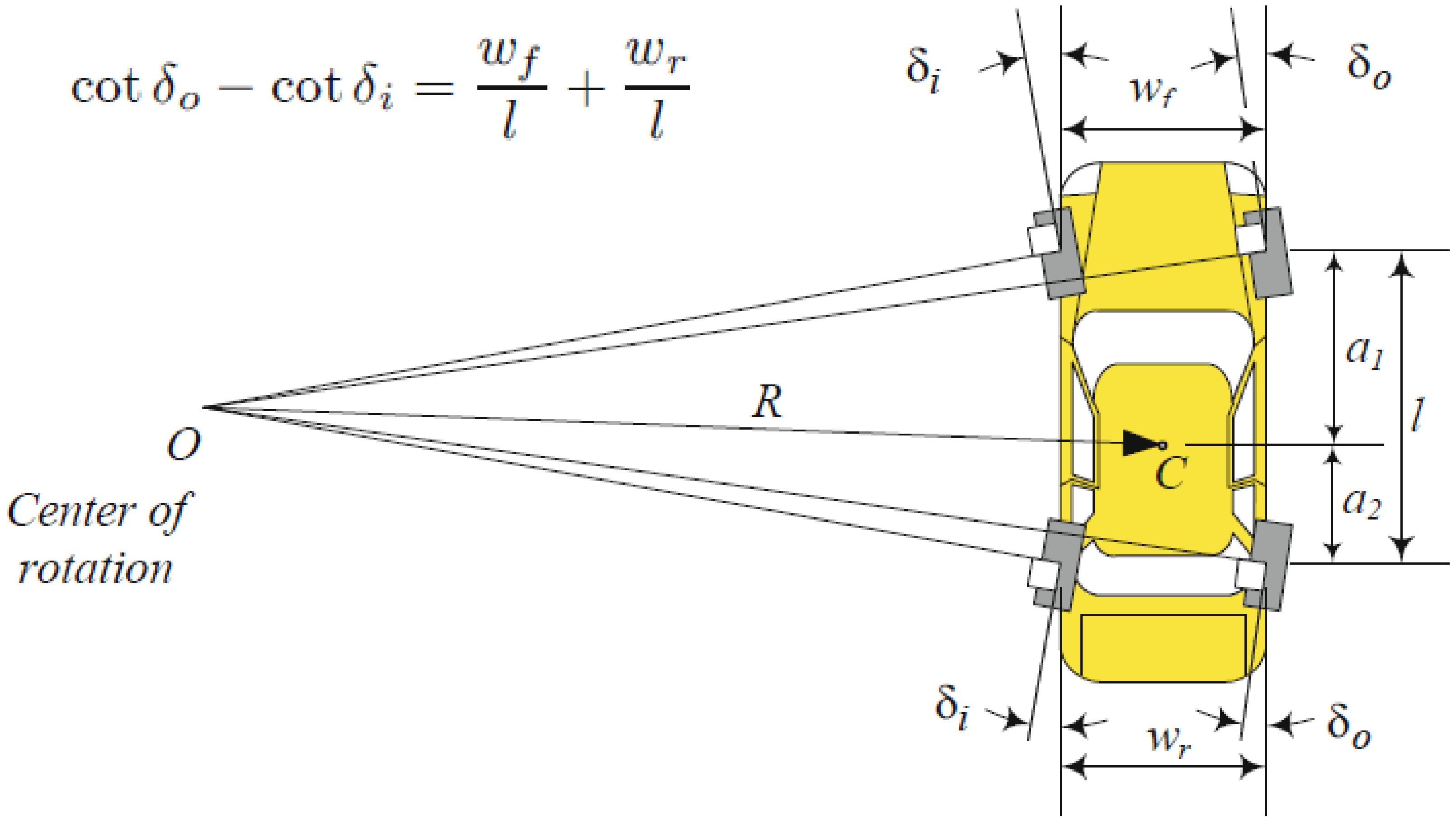
FWS (Front Wheel Steering) vehicle:

$$x_O = -a_2 \qquad y_O = \frac{1}{2}w_f + \frac{l}{\tan\delta_{if}}$$



Symmetric four-wheel steering system

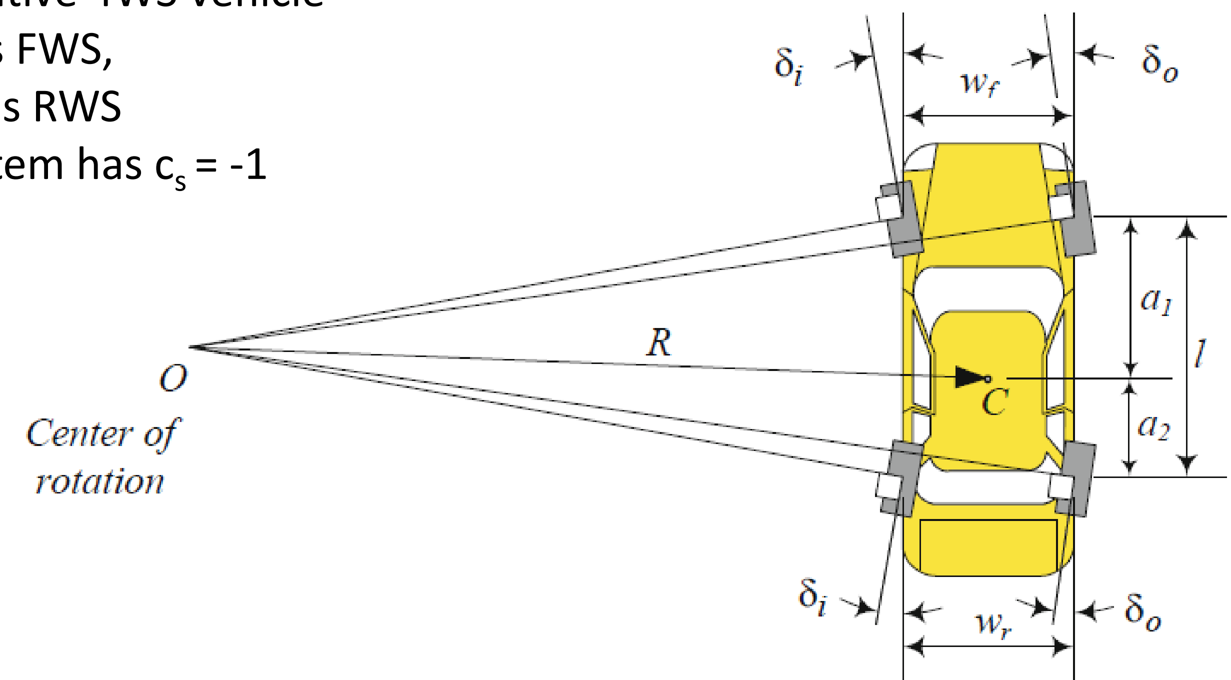
$$\cot\delta_o - \cot\delta_i = \frac{w_f}{l} + \frac{w_r}{l}$$



Four-wheel steering factor

$$c_s = \frac{c_2}{c_1} = \frac{w_r}{w_f}\frac{\cot\delta_{fr} - \cot\delta_{fl}}{\cot\delta_{rr} - \cot\delta_{rl}}$$

c_s is negative for a negative 4WS vehicle
c_s is positive for a positive 4WS vehicle
When $c_s = 0$, the car is FWS,
When $c_s = \infty$, the car is RWS
A symmetric 4WS system has $c_s = -1$



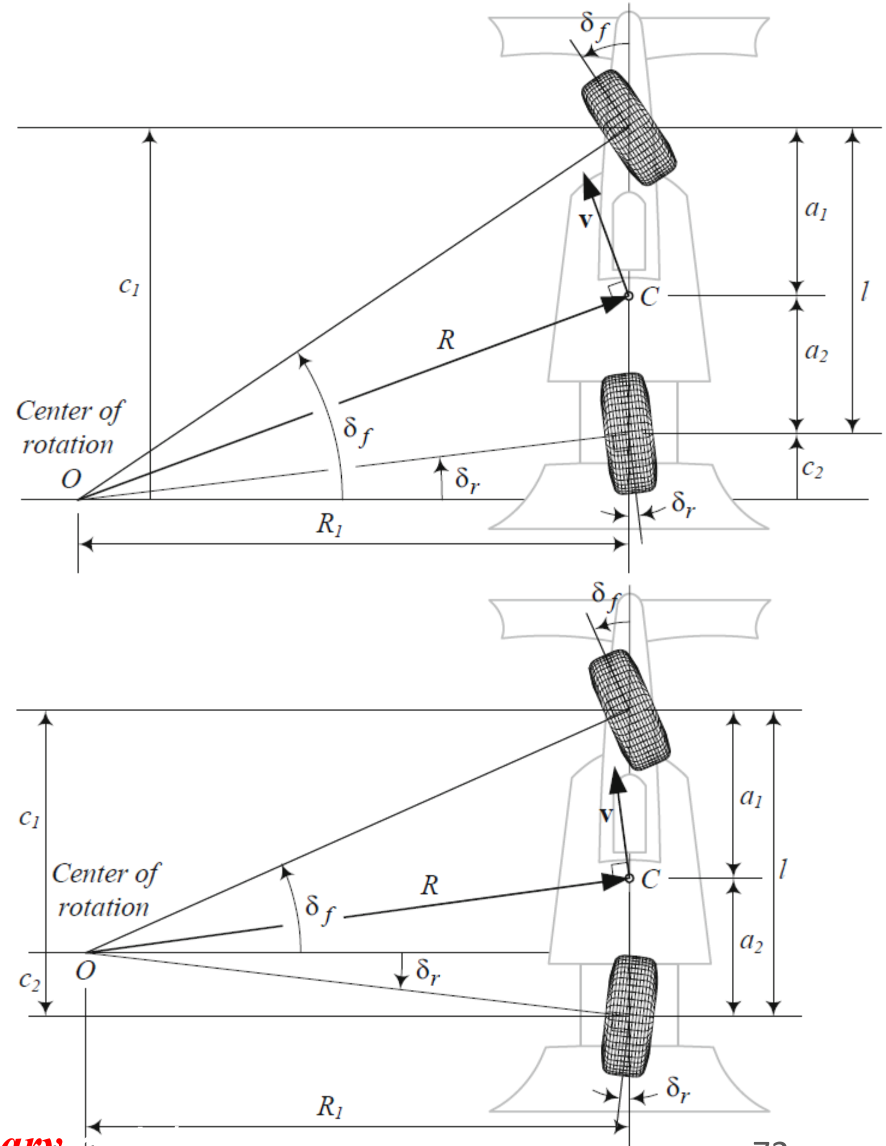
Turning radius

Positive or Negative 4WS:

$$R^2 = (a_2 + c_2)^2 + R_1^2$$

$$\cot \delta_f = \frac{R_1}{c_1} = \frac{1}{2}\left(\cot \delta_{if} + \cot \delta_{of}\right)$$

$$R = \sqrt{(a_2 + c_2)^2 + c_1^2 \cot^2 \delta_f}$$



Curvature radius

Road path a function Y= F(X), in a global coordinate frame

The radius of curvature

$$R_\kappa = \frac{\left(1 + Y'^2\right)^{3/2}}{Y''}$$

$$Y' = \frac{dY}{dX} \qquad Y'' = \frac{d^2Y}{dX^2}$$

$$\kappa = \frac{Y''}{(1 + Y'^2)^{3/2}}$$

$$R_\kappa = \frac{1}{\kappa} = \frac{\left(1 + Y'^2\right)^{3/2}}{Y''} = \frac{\left(\dot{X}^2 + \dot{Y}^2\right)^{3/2}}{\ddot{Y}\dot{X} - \ddot{X}\dot{Y}}$$

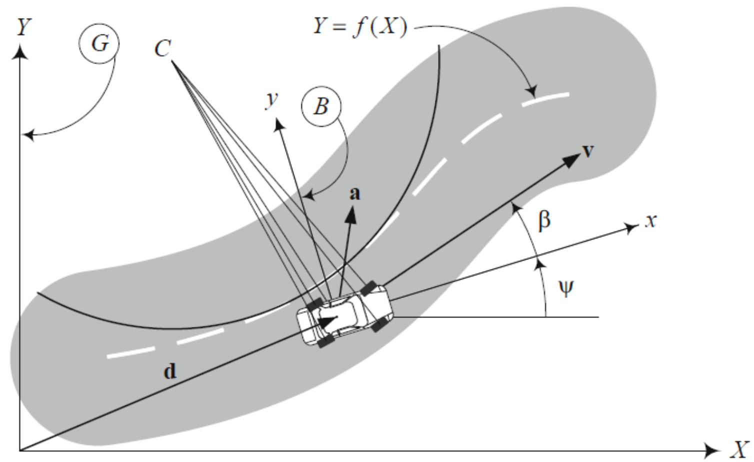
Curvature radius

Path a function X= X(t), Y = Y(t) in a global coordinate frame

In the car coordinate

$$
\begin{aligned}
x_C &= (X_C - X)\cos\psi + (Y_C - Y)\sin\psi \\
&= \left(\dot{X}\sin\psi - \dot{Y}\cos\psi\right)\frac{\dot{X}^2 + \dot{Y}^2}{\ddot{Y}\dot{X} - \ddot{X}\dot{Y}}
\end{aligned}
$$

$$
\begin{aligned}
y_C &= (Y_C - Y)\cos\psi - (X_C - X)\sin\psi \\
&= \left(\dot{X}\cos\psi + \dot{Y}\sin\psi\right)\frac{\dot{X}^2 + \dot{Y}^2}{\ddot{Y}\dot{X} - \ddot{X}\dot{Y}}
\end{aligned}
$$



In the global coordinate

$$
\begin{aligned}
X_C &= X - \frac{\dot{X}^2 + \dot{Y}^2}{\ddot{Y}\dot{X} - \ddot{X}\dot{Y}}\dot{Y} \\
Y_C &= Y + \frac{\dot{X}^2 + \dot{Y}^2}{\ddot{Y}\dot{X} - \ddot{X}\dot{Y}}\dot{X}
\end{aligned}
$$